Introduction to the current dipole

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Prelude

What’s the problem
You want to understand this...
...by using this...
...and physicists ask you:

- Which part of

\[
\left( \frac{\mu_0}{27\pi^3} \right)^{3/2} \oint_\Omega \nabla \times \nabla \times \nabla \times \vec{B} \cdot d\mathbf{S} = -\frac{\partial}{\partial t} \iiint \nabla (\nabla \cdot \vec{E}) + J / 32
\]

you don’t understand?
Vector fields

Concepts of vector analysis
Example: flow of a fluid

- Take a snapshot:

- Velocity depends on position:

\[ \vec{v} = \vec{v}(x,y,z) \]
Vector field: a vector at each location
More descriptive representation

- Streamlines: density is proportional to strength
Or better with both…
Divergence of vector field

- Divergence: (local net flow out / unit volume)
- Flow in is negative flow out
Divergence visualized
Curl of vector field

- Curl: (local net circulation/unit area)
- Curl is a vector: circulation about a direction

Positive curl

Negative curl
Curl visualized
Why divergence, why curl?

- Maxwell’s eq’s are written in terms of div and curl
  - Divergence describes the sources and sinks of the field
  - Curl describes the circulation of the field

- In general, div and curl describe fully a vector field
  - This is called Helmholtz’s decomposition
  - “Fundamental theorem” of vector calculus
Electro- and magnetostatics

Separation of Electric and Magnetic Fields
Maxwell’s equations, here they are!

\[
\begin{align*}
\text{div}(\vec{E}) &= \rho / \epsilon_0 \\
\text{curl}(\vec{E}) &= -\frac{\Delta \vec{B}}{\Delta t} \\
\text{div}(\vec{B}) &= 0 \\
\text{curl}(\vec{B}) &= \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{\Delta \vec{E}}{\Delta t}
\end{align*}
\]

\(\vec{E}\) = electric field  \\
\(\vec{B}\) = magnetic field  \\
\(\rho\) = charge density  \\
\(\vec{J}\) = current density  \\
\(\mu_0, \epsilon_0\) = constants
If static situation: \( \frac{\Delta \vec{B}}{\Delta t} = \frac{\Delta \vec{E}}{\Delta t} = 0 \), and \( \rho, \vec{J} \) time-independent

\[
\text{div}(\vec{E}) = \rho / \epsilon_0
\]
\[
\text{curl}(\vec{E}) = 0
\]

depend only on \( \rho \) and \( \vec{E} \)

\[
\text{div}(\vec{B}) = 0
\]
\[
\text{curl}(\vec{B}) = \mu_0 \vec{J}
\]

depend only on \( \vec{J} \) and \( \vec{B} \)
Electric field of a charged particle

- The field decays as
  \[ E \propto \frac{1}{r^2} \]

- Orientation:
  - From the source (+)
  - To the source (-)

- This generates force
  \[ \vec{F} = q\vec{E} \]
Electric field of a charged particle (cont.)

- Field line ("streamline") representation
Electric field of a charged particle (cont)

- Things blow up when $r \to 0$
Superposition: fields of two sources add up
Superposition (Field lines)
The divergence of $\vec{E}$ (1st Static Maxwell)

- Divergence is nonzero only where the charges are!
The curl of $\vec{E}$ is zero everywhere.

Curl of $\vec{E}$ (2\textsuperscript{nd} Static Maxwell)