Why do we need Signal Processing N How do we conceptualize it?

Adrian KC Lee, ScD
Research Fellow, Martinos Center

Why.N.How Tutorial Series
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Format
- Section 1
  - Time / Frequency (Duality)
  - Linear Time-Invariant Systems
  - Issues with Sampling (Aliasing)
- Section 2
  - Windowing / Filter Design
  - Phase information
  - K-space / Image Processing Examples

“Why.N.How Signal Processing”

Why spectral analysis?

Frequency (Spectral) Analysis
- Bio-signal is intrinsically rhythmic:
  - Brain rhythm (alpha; beta; theta etc.)
  - Heart beat / Respiratory cycle
- Filtering = Scaling in Frequency
  - Artifact rejection (60 Hz AC power line)
  - Lowpass / Highpass / Bandpass filters
- Same fundamental concepts in MRI:
  - K-space = spatial frequency (2D instead of 1D)
  - Spatial Smoothing = Lowpass filtering in space

Section 1
Time-frequency (Duality)
Convolution / Filtering
Sampling / Aliasing
Are you smarter than a 5th grader?

5th Grade Math Question: Evaluate
\[(1 + z)(5z^5 - 5z^3 + 3z^2 - 3z^2 + 1)\]
\[= 1 + 5z^5 - 5z^3 + 3z^2 - 3z^2 + 1z - 1\]
\[= 5z^6 + 0z^5 - 2z^4 + 0z^3 - 2z^2 + 0z - 1\]
Assumptions & Caveats
- Linear and Time-Invariant System
- Saturation of a system violates linearity!
- Obey Principle of Superposition
- Beware of Absolute values: $|3-5| = |-2| = 2 \neq |3|-|-5| = -2$
- Adaptation violates time-invariance assumption!

Aliasing
- Are you sampling adequately?

Nyquist Theorem: Sampling at least $2F_s$

MRI Aliasing / Wrap-around
http://www.e-mri.org/quality-artifacts/aliasing.html
Section 2
Windowing / Filter Design
Trade-offs & considerations
K-space / Image Processing

Windowing
✓ Frequency "spillage" vs. bandwidth

Complex number / Z-plane / DTFT
✓ 2 parts to a complex number:
  ✓ $i^2 = -1$
  ✓ Real and Imaginary ($3 + 4i$)
  ✓ Magnitude and Phase ($5e^{j\phi}$)

Discrete Time Fourier Transform
$$X(e^{\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Why do we care about the phase?
✓ Be careful with non-linear phase (dispersion effect)!

Filtering
Full-Width Half-Max (FWHM)
Filtered Signal (LPF Speech)

Filtered Signal (BPF Speech)

Filtered Signal (HPF Speech)

What is k-space?

Filtering in k-space

Image Signal Processing