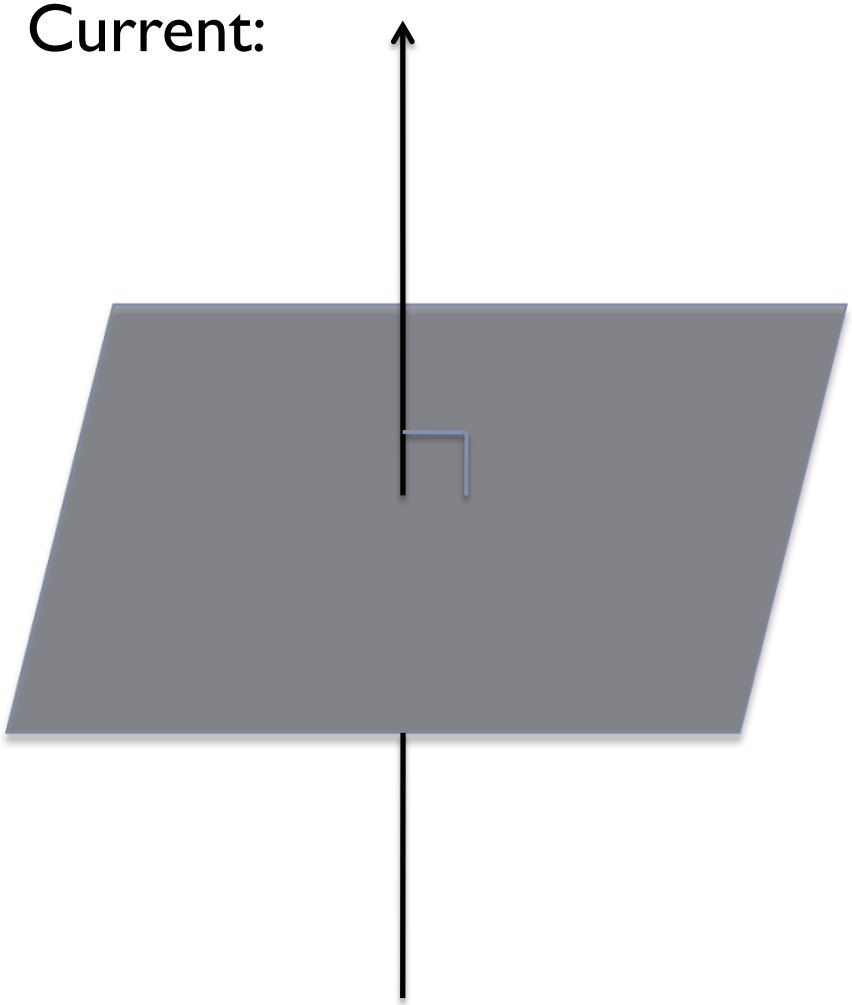
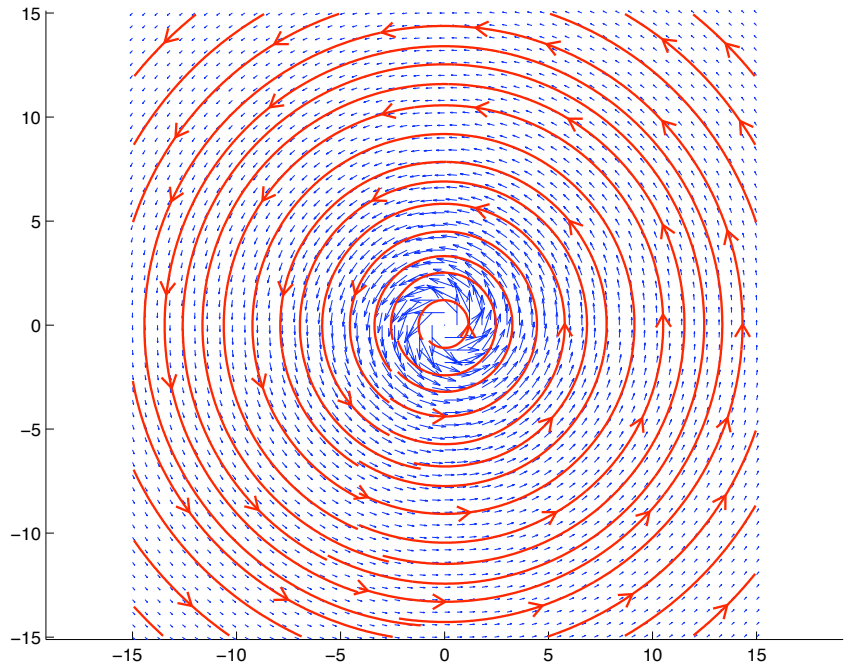
Magnetic field of a long thin wire

Current:

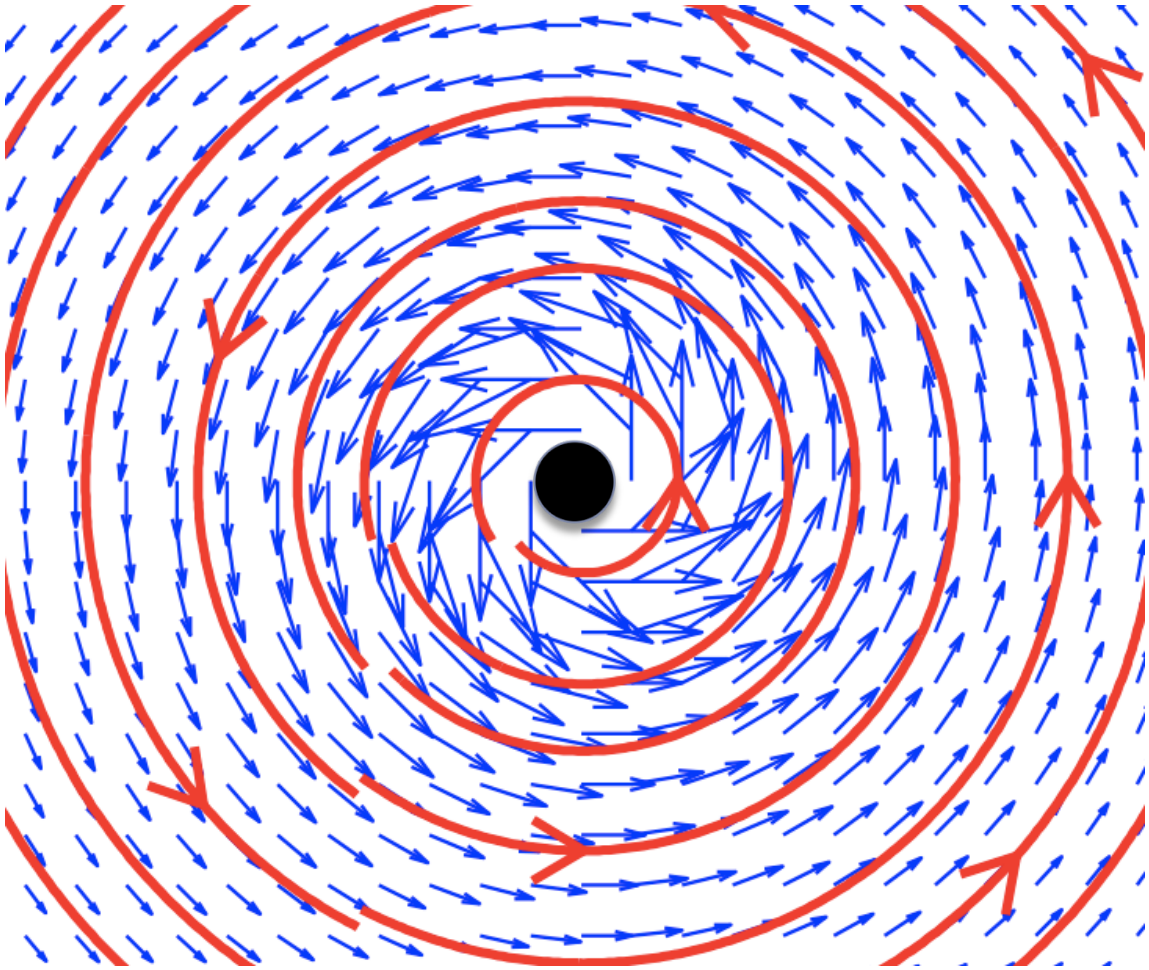


Magnetic field in plane:



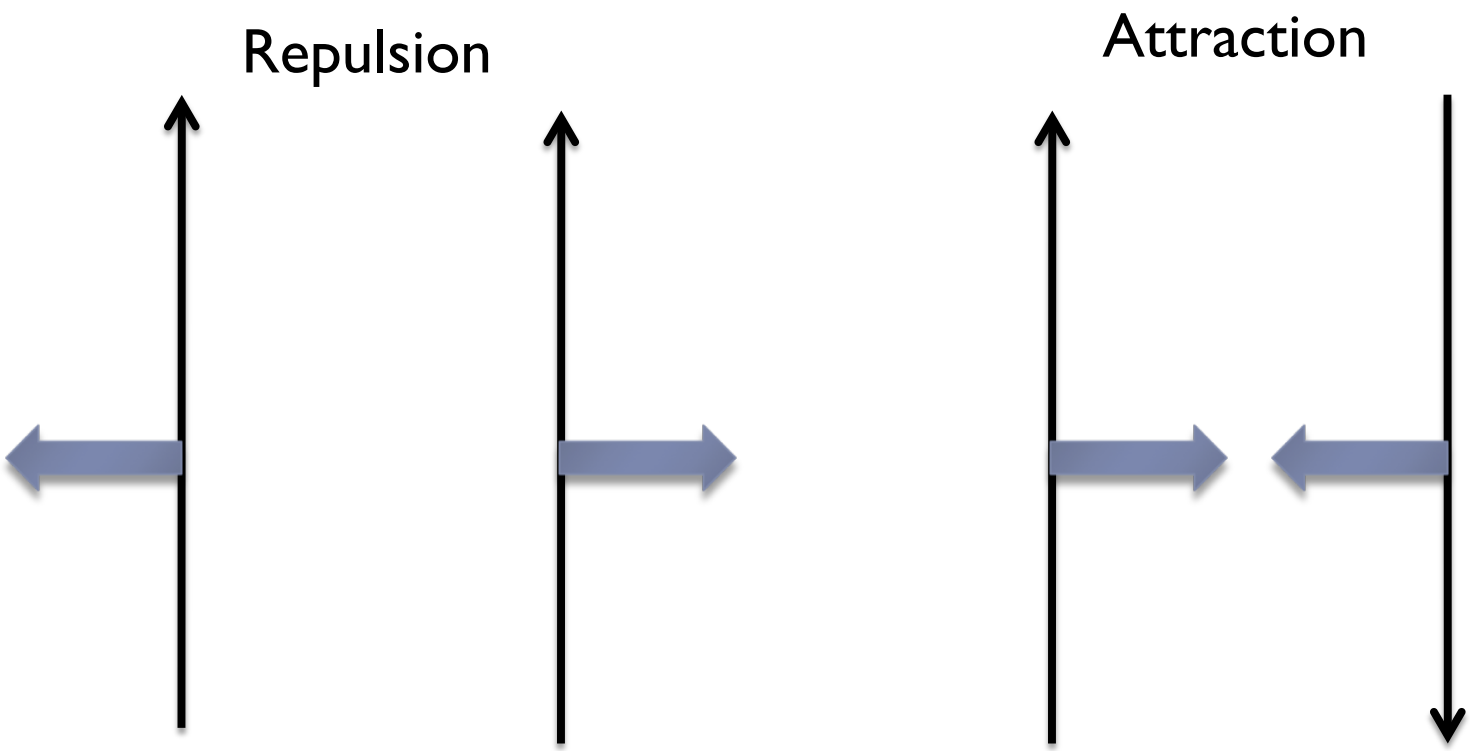
Magnetic field of a long thin wire (cont)

The magnetic field drops off as $1/r$



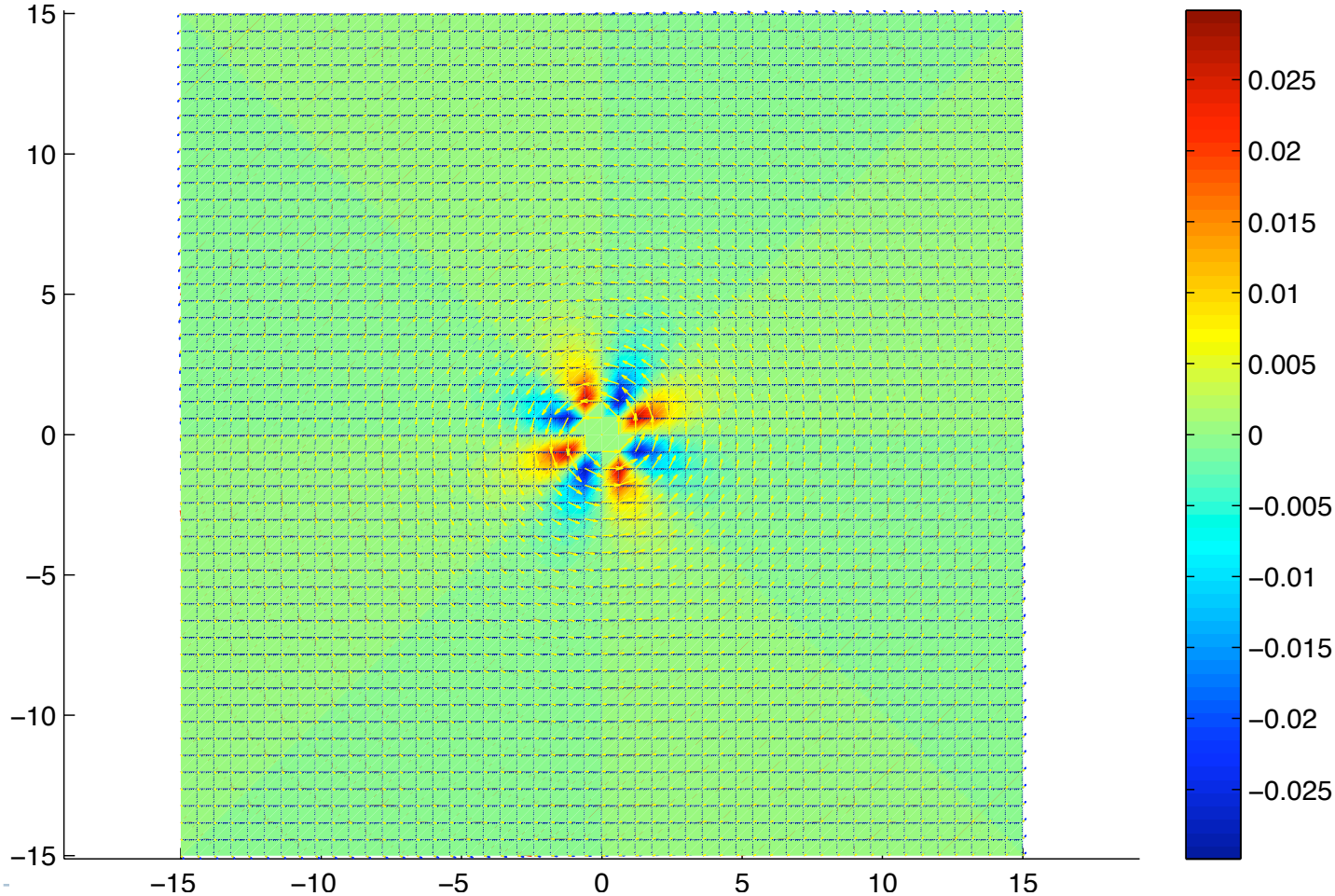
Magnetic forces between wires

- There exists magnetic forces between to wires:



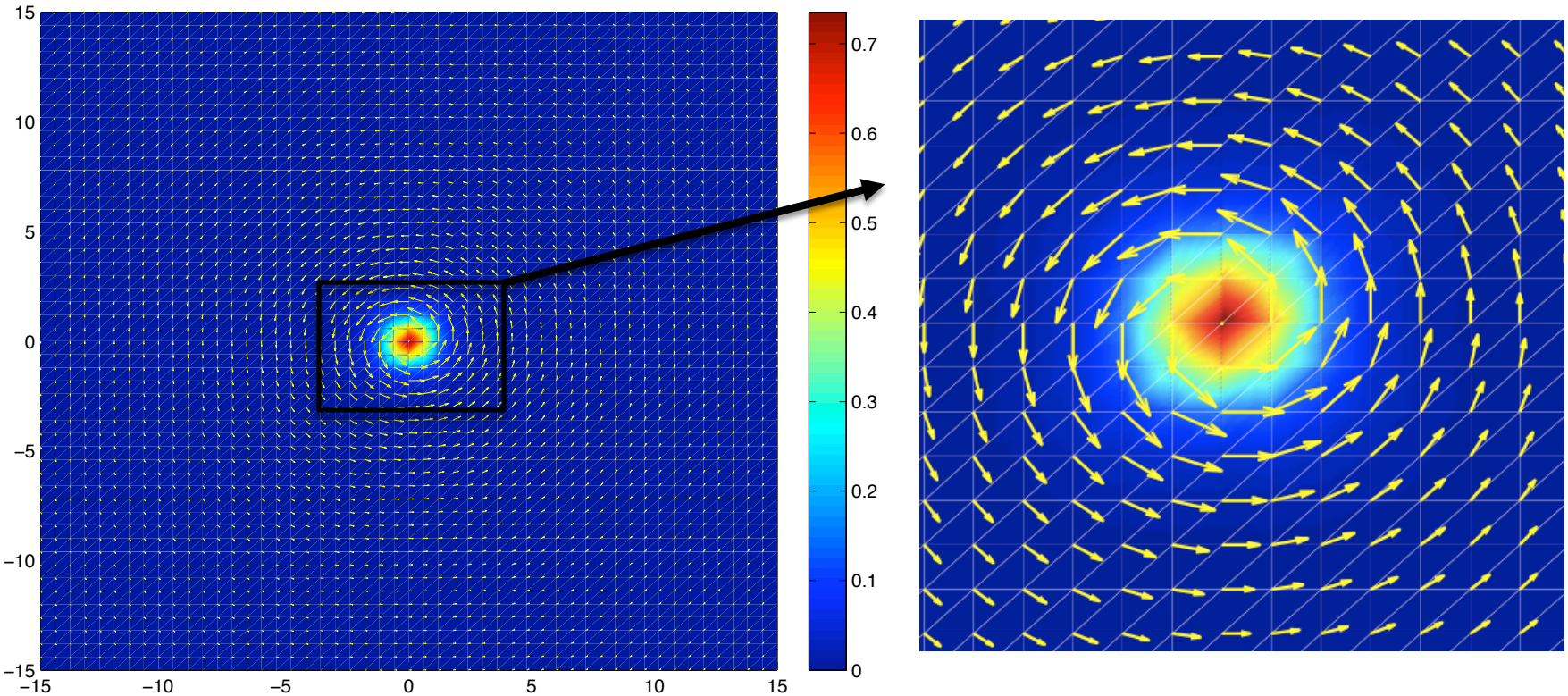
Divergence of $\vec{B}$ (3rd Static Maxwell)

The divergence of $\vec{B}$ is zero everywhere (well…)



The curl of $\vec{B}$ (4th Static Maxwell)

The curl is nonzero only where the current is!



Static Maxwell's summary

$\mathrm{div}(\vec{E}) = \rho / \epsilon_0 \quad \longleftrightarrow$ Charges are sources (and sinks) of $\vec{E}$

$\mathrm{curl}(\vec{E}) = 0 \quad \longleftrightarrow$ Nothing makes $\vec{E}$ circulate

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$\mathrm{div}(\vec{B}) = 0 \quad \longleftrightarrow$ There are no sources (or sinks) of $\vec{B}$

$\mathrm{curl}(\vec{B}) = \mu_0 \vec{J} \quad \longleftrightarrow$ Current makes $\vec{B}$ circulate

Quasi-static Maxwell's

And the proverbial current dipole!

Quasi-static = almost static

- The separation of $\vec{E}$ and $\vec{B}$ is very convenient
- Assume that everything changes slowly
- Static equations are solved for each time point
- Radio frequency ~ Million Hertz
- Brain frequency < Hundred Hertz
- We ignore effects of electromagnetic waves

There's just one more thing…

- For arbitrary vector field:

$$\mathrm{div}(\mathrm{curl}(\vec{V})) = 0$$

- In statics, the divergence of the current vanishes:

$$\vec{J} = 1/\mu_0 \mathrm{curl}(\vec{B}) \quad \Rightarrow \quad \mathrm{div}(\vec{J}) = 1/\mu_0 \mathrm{div}(\mathrm{curl}(\vec{B})) = 0$$

- Quasi-static currents must form closed loops!

$$\mathrm{div}(\vec{J}) = 0$$

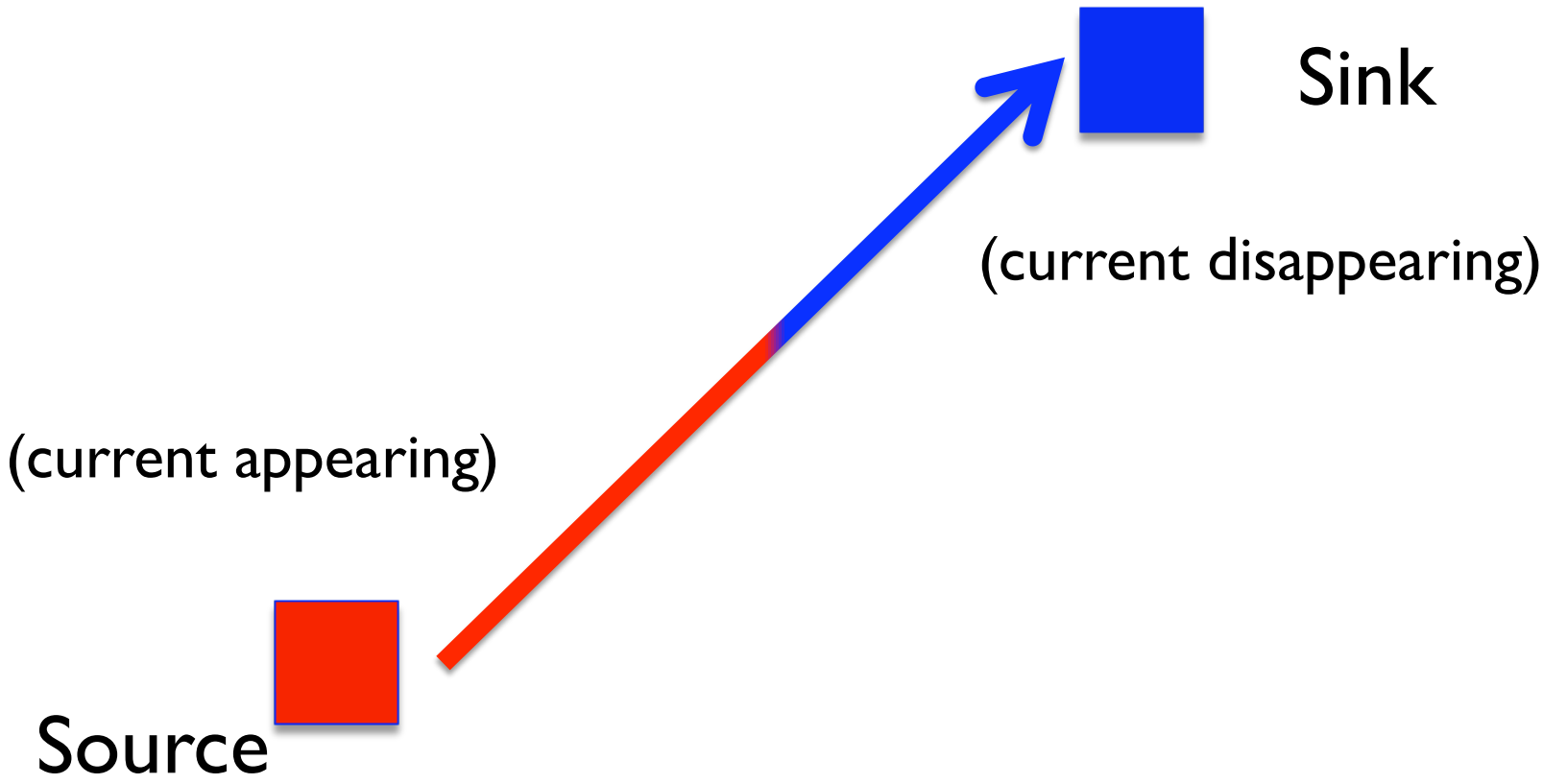
Oh, and yet another thing…

- We must agree upon a length scale
  - Currents, etc. are to be measured on this scale
  - All scales/measurements are approximate
  - Suitable scale depends on application

- Let us assume "neuroimaging" scale: 1mm

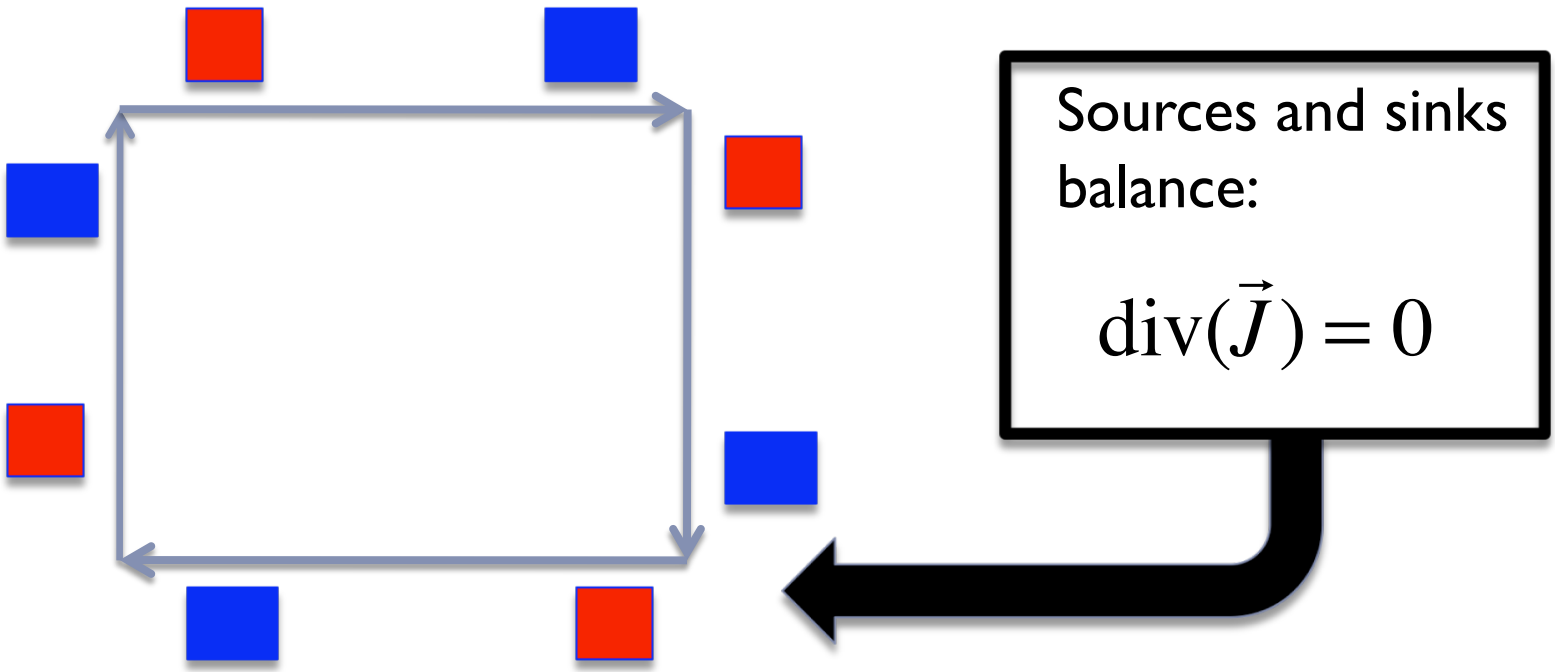
So what is the current dipole?

- For our purposes, it is just a piece of current:



What is the current dipole good for?

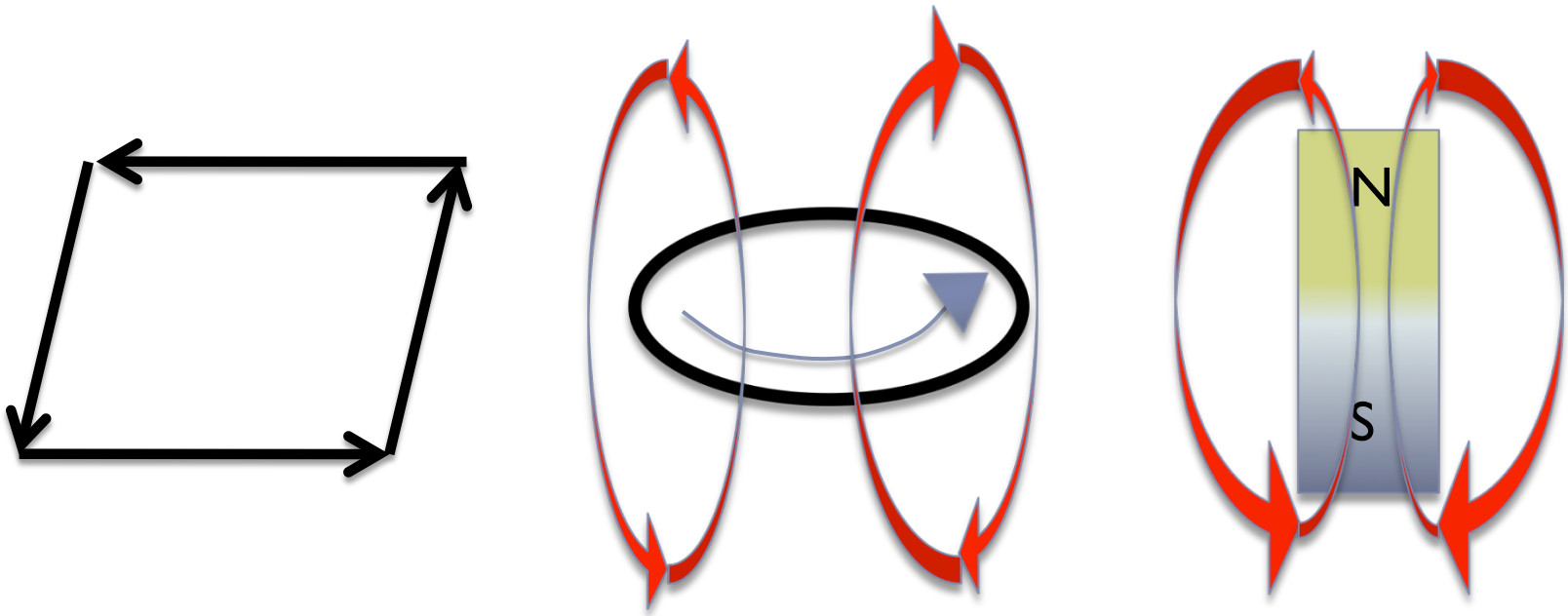
- I can make more complicated currents by adding dipoles:



- The fields of the more complicated currents can be found as superposition of dipoles!

Current dipole is NOT same as magnetic dipole

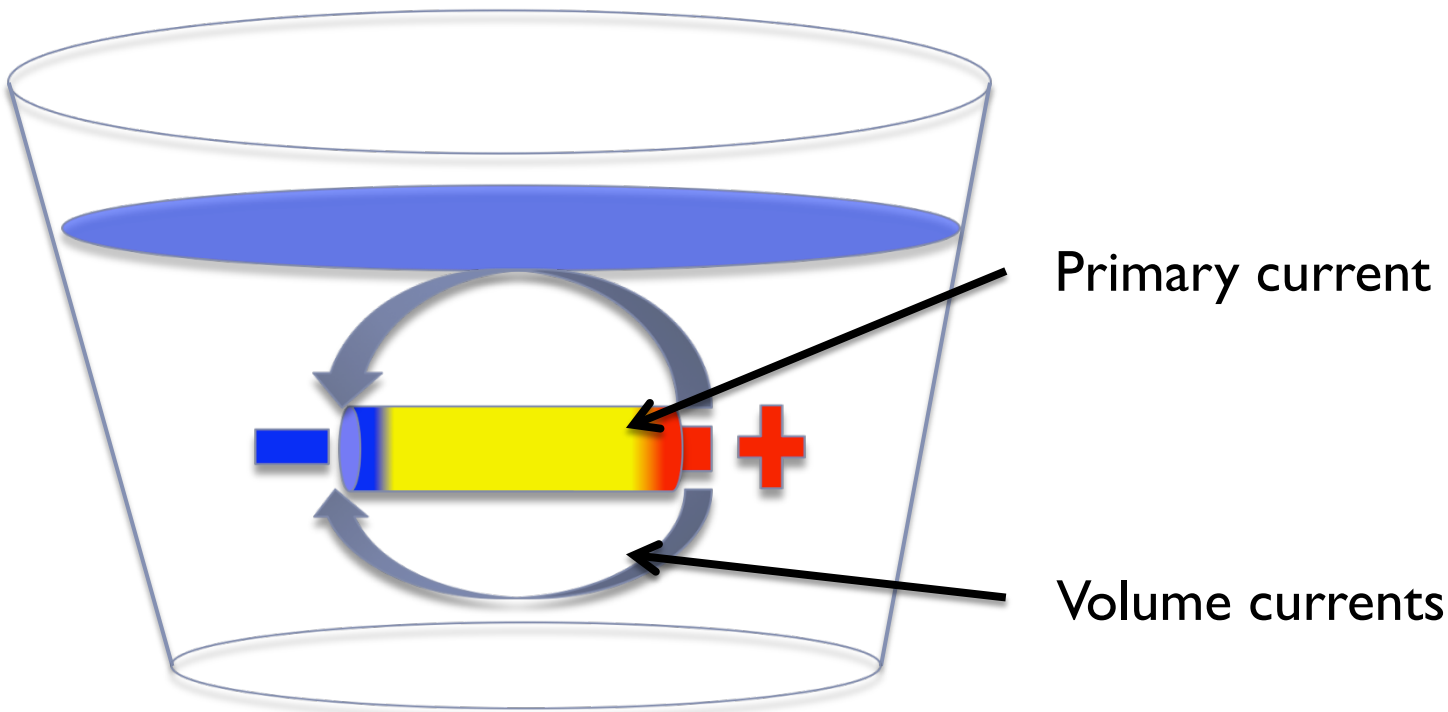
- Magnetic dipole is equivalent to a small current loop



- Of course, this can be approximated with current dipoles

What is the dipole in real life?

- To get going, we must have a current source.
- What happens if a battery gets into salt water?
  - Currents in the water flow to close the circuit.



Inside and outside the battery

- Inside and outside must be separated by a shield
  - Inside, the charges are "lifted" from − to +

- Inside: we model the current by a single dipole

$$\text{primary current} = \vec{J}_p = \text{dipole}$$

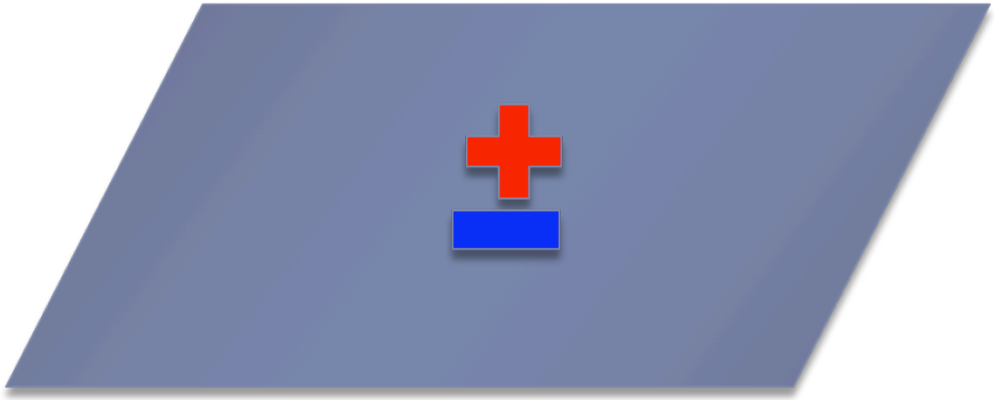
- Outside: We assume Ohm's law

$$\text{volume current} = \vec{J}_v = \sigma \vec{E}$$

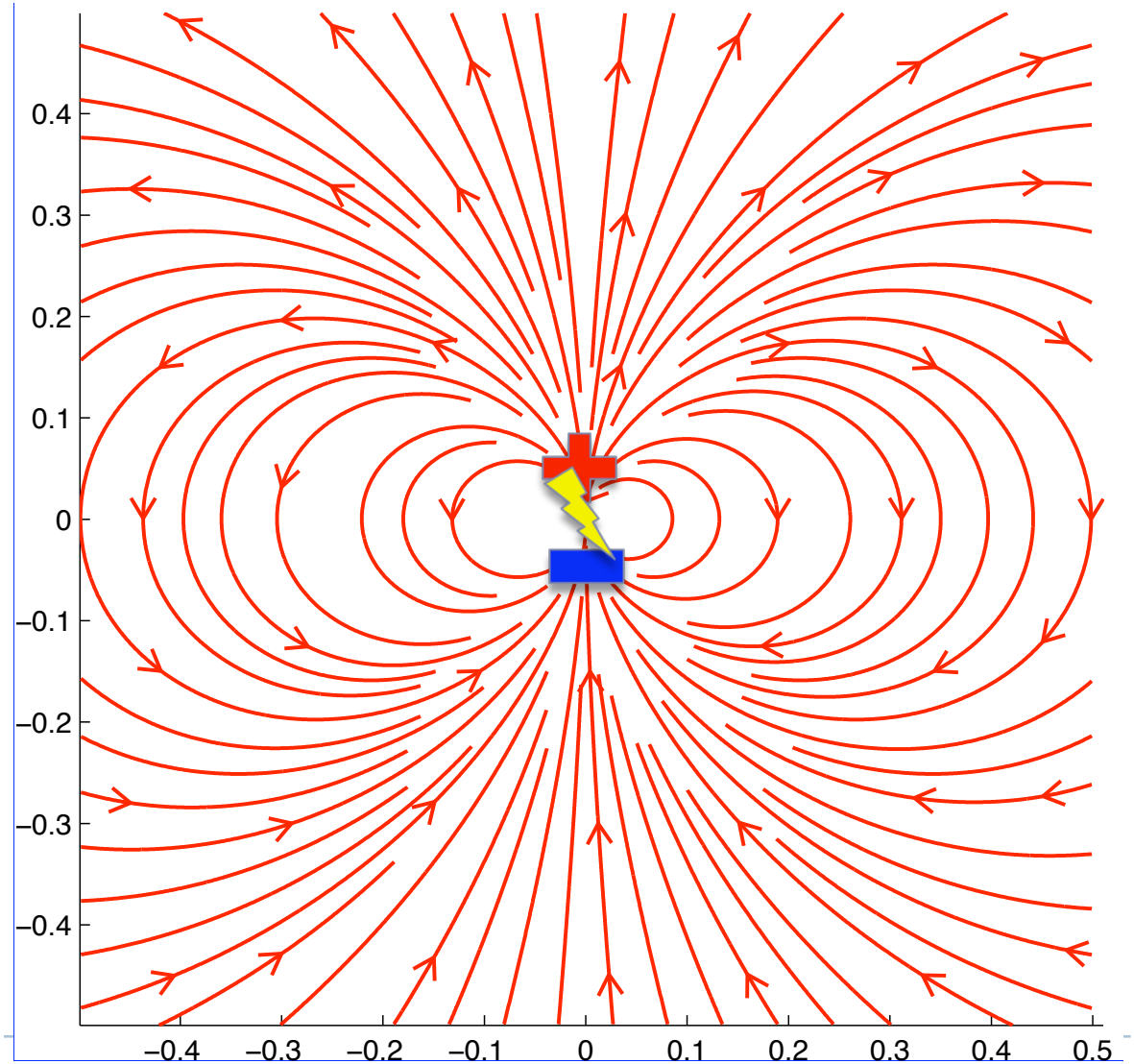
(current inside)+(current outside)=closed "loop"

How to find a matching outside current for a given dipole?

- How to find the electric field of a dipole?
- Let's assume a 2D homogeneous conductor.
- The E-field of current dipole is field is equivalent to two closely spaced "static" charges:



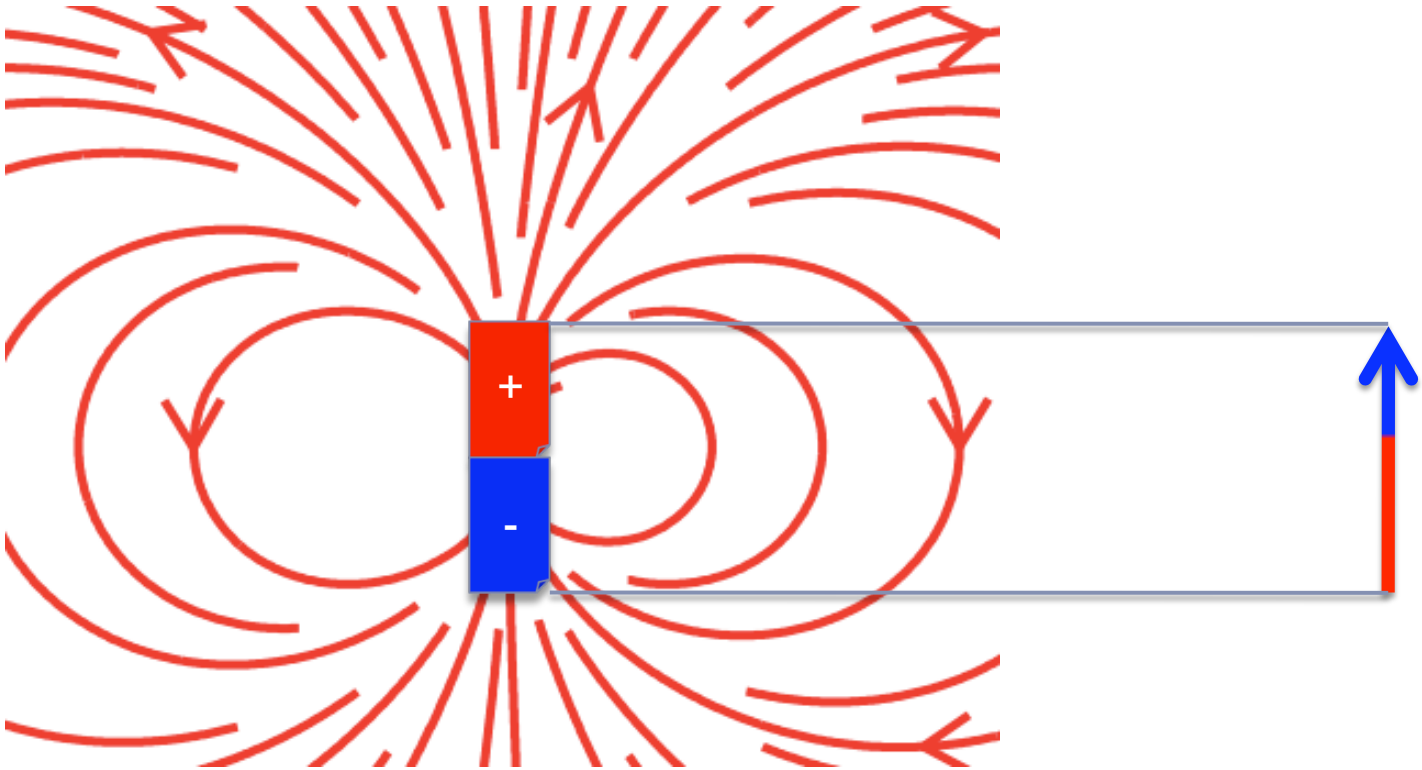
The E-field of the current dipole



The total current has divergence zero

Volume (outside)

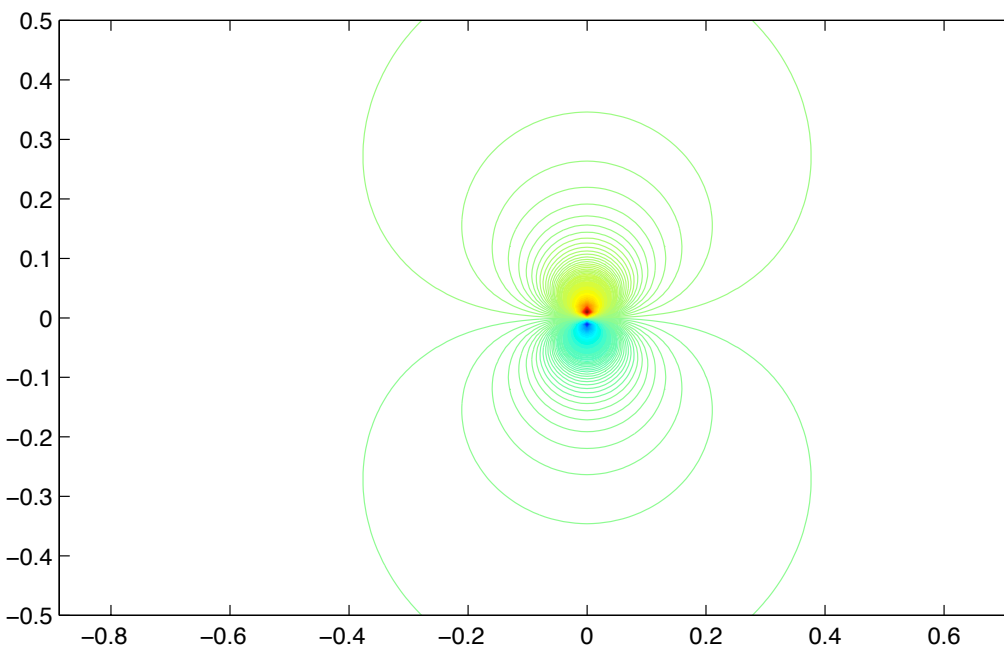
Primary (inside)

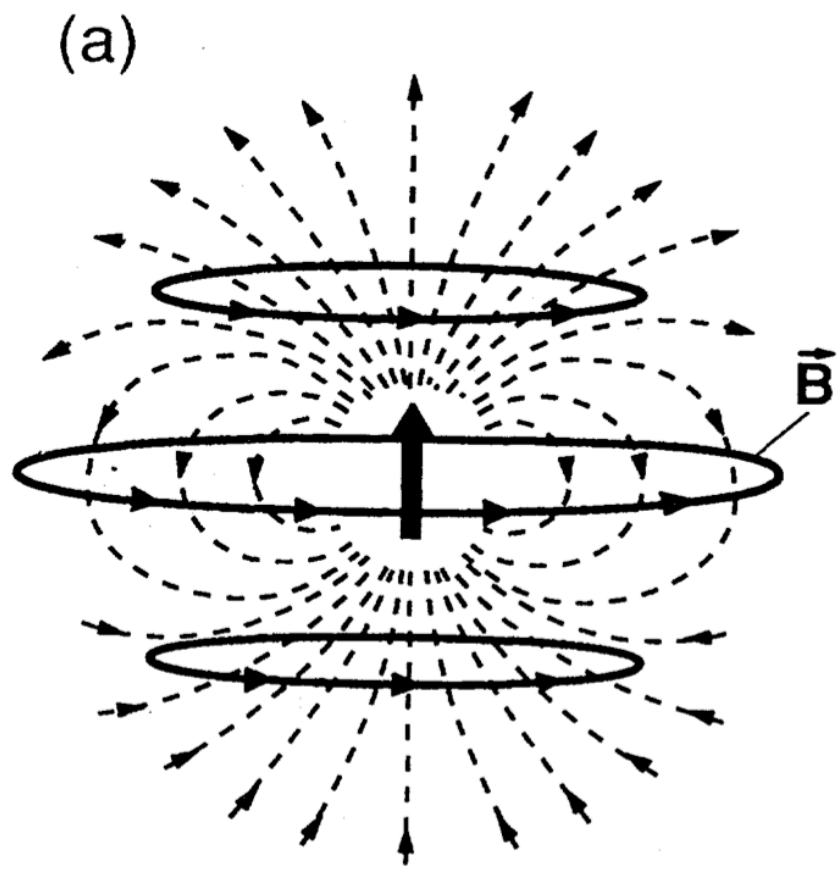


We know everything now!

- We know potential, all currents, we can calculate magnetic fields…

The "EEG" (potential)





Back to earth: realistic models

- Head is not a homogeneous 2D conductor
- Conductivity boundaries (skin, skull, scalp) important
  - Need different MRI's to find these

- Principles remain the same:
  - Assume battery -> find volume currents

- Volume currents are more complicated

Source localization

Briefest ever

Dipole fitting & distributed source models

- Localize the "neural batteries" -> activation

- Dipole fitting
  - Assume that there are a couple of neural batteries
  - Find locations which best explain measured EEG/MEG
- Distributed source models
  - Assume a distribution of batteries throughout brain
  - "Doomed" to get confused: non-uniqueness!
- E/MEG are measured far from the sources -> blurring

And finally…

Return of the evil physicist

Don't forget The Truth!

$$\nabla \cdot \vec{E} = \rho / \epsilon_0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

Thanks for listening!