Magnetic field of a long thin wire

Current:

Magnetic field in plane:
Magnetic field of a long thin wire (cont)

The magnetic field drops off as $\frac{1}{r}$
There exists magnetic forces between two wires:

- **Repulsion**

- **Attraction**
Divergence of $\vec{B}$ (3rd Static Maxwell)

The divergence of $\vec{B}$ is zero everywhere (well…)

The graph shows a divergence map with a color scale ranging from 0 to 0.025, indicating the divergence values at different points in the space.
The curl of $\vec{B}$ (4\textsuperscript{th} Static Maxwell)

The curl is nonzero only where the current is!
Static Maxwell’s summary

\[ \text{div}(\vec{E}) = \rho / \epsilon_0 \quad \iff \quad \text{Charges are sources (and sinks) of } \vec{E} \]

\[ \text{curl}(\vec{E}) = 0 \quad \iff \quad \text{Nothing makes } \vec{E} \text{ circulate} \]

\[ \text{div}(\vec{B}) = 0 \quad \iff \quad \text{There are no sources (or sinks) of } \vec{B} \]

\[ \text{curl}(\vec{B}) = \mu_0 \vec{J} \quad \iff \quad \text{Current makes } \vec{B} \text{ circulate} \]
Quasi-static Maxwell’s

And the proverbial current dipole!
Quasi-static = almost static

- The separation of $\vec{E}$ and $\vec{B}$ is very convenient
- Assume that everything changes slowly
- Static equations are solved for each time point
- Radio frequency $\sim$ Million Hertz
- Brain frequency $<$ Hundred Hertz
- We ignore effects of electromagnetic waves
There’s just one more thing…

- For arbitrary vector field:

\[ \text{div}(\text{curl}(\vec{V})) = 0 \]

- In statics, the divergence of the current vanishes:

\[ \vec{J} = \frac{1}{\mu_0} \text{curl}(\vec{B}) \quad \Rightarrow \quad \text{div}(\vec{J}) = \frac{1}{\mu_0} \text{div}(\text{curl}(\vec{B})) = 0 \]

- Quasi-static currents must form closed loops!

\[ \text{div}(\vec{J}) = 0 \]
Oh, and yet another thing...

- We must agree upon a length scale
  - Currents, etc. are to be measured on this scale
  - All scales/measurements are approximate
  - Suitable scale depends on application

- Let us assume “neuroimaging” scale: 1 mm
So what is the current dipole?

- For our purposes, it is just a piece of current:
  - Source (current appearing)
  - Sink (current disappearing)
What is the current dipole good for?

- I can make more complicated currents by adding dipoles:

\[ \text{Sources and sinks balance: } \text{div}(\vec{J}) = 0 \]

- The fields of the more complicated currents can be found as superposition of dipoles!
Current dipole is NOT same as magnetic dipole

- Magnetic dipole is equivalent to a small current loop

- Of course, this can be approximated with current dipoles
What is the dipole in real life?

- To get going, we must have a current source.
- What happens if a battery gets into salt water?
  - Currents in the water flow to close the circuit.
Inside and outside the battery

- Inside and outside must be separated by a shield
  - Inside, the charges are “lifted” from \([-\) to \([+] \)

- Inside: we model the current by a single dipole
  \[ \text{primary current} = \bar{J}_p = \text{dipole} \]

- Outside: We assume Ohm’s law
  \[ \text{volume current} = \bar{J}_v = \sigma \bar{E} \]

\[(\text{current inside})+(\text{current outside})=\text{closed ”loop”}\]
How to find a matching outside current for a given dipole?

- How to find the electric field of a dipole?

- Let's assume a 2D homogeneous conductor.

- The E-field of current dipole is field is equivalent to two closely spaced “static” charges:
The E-field of the current dipole
The total current has divergence zero

Volume (outside)  Primary (inside)
We know everything now!

- We know potential, all currents, we can calculate magnetic fields…

The “EEG” (potential)
Back to earth: realistic models

- Head is not a homogeneous 2D conductor
- Conductivity boundaries (skin, skull, scalp) important
  - Need different MRI’s to find these

- Principles remain the same:
  - Assume battery -> find volume currents

- Volume currents are more complicated
Source localization

Briefest ever
Dipole fitting & distributed source models

- Localize the “neural batteries” -> activation

- Dipole fitting
  - Assume that there are a couple of neural batteries
  - Find locations which best explain measured EEG/MEG

- Distributed source models
  - Assume a distribution of batteries throughout brain
  - “Doomed” to get confused: non-uniqueness!

- E/MEG are measured far from the sources -> blurring
And finally…

Return of the evil physicist
Don’t forget The Truth!

\[ \nabla \cdot \mathbf{E} = \rho / \epsilon_0 \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

\[ \nabla \cdot \mathbf{B} = 0 \]

\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} \]
Thanks for listening!

Questions?