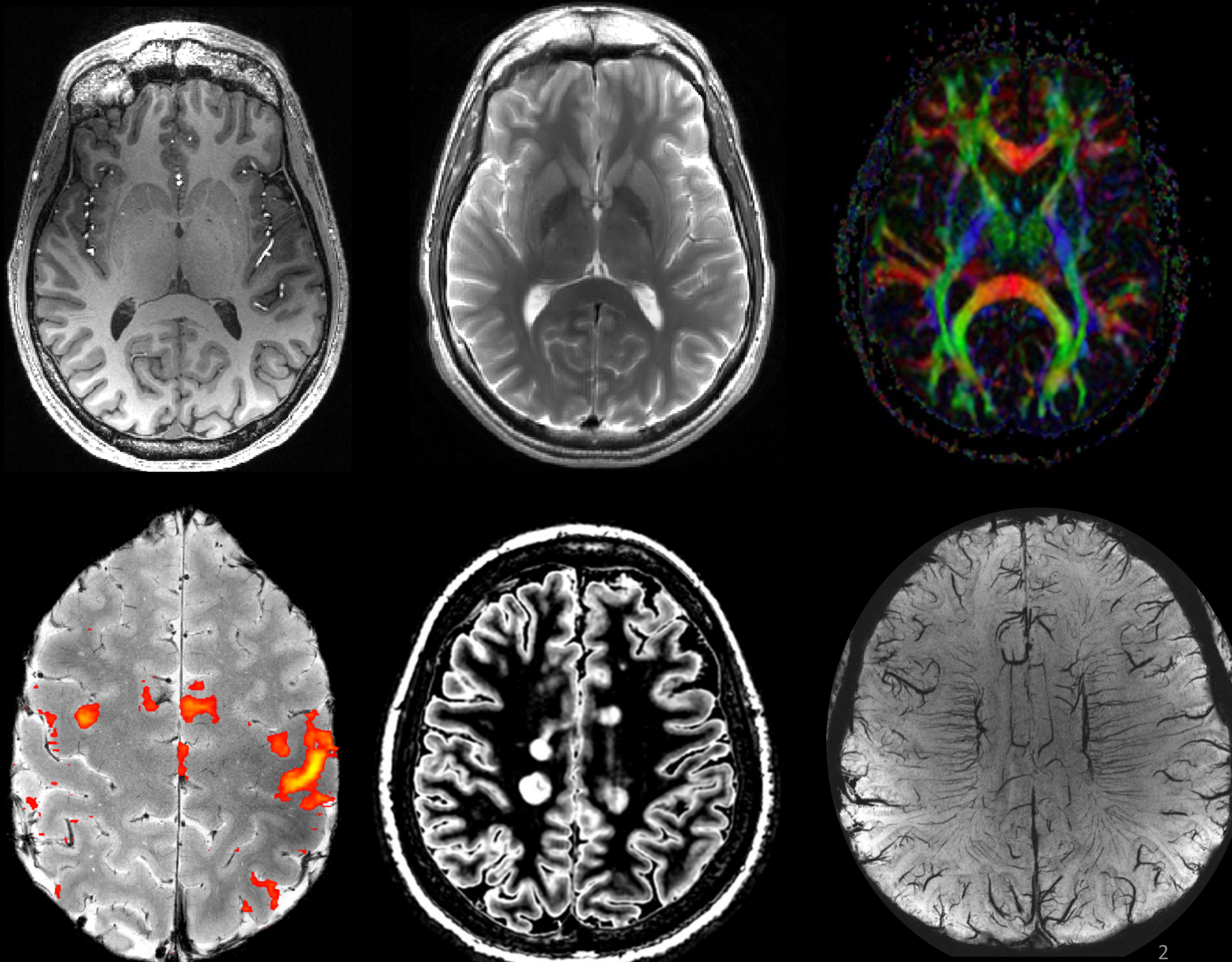
Why and How: MRI Physics

Patrick McDaniel

10/10/19

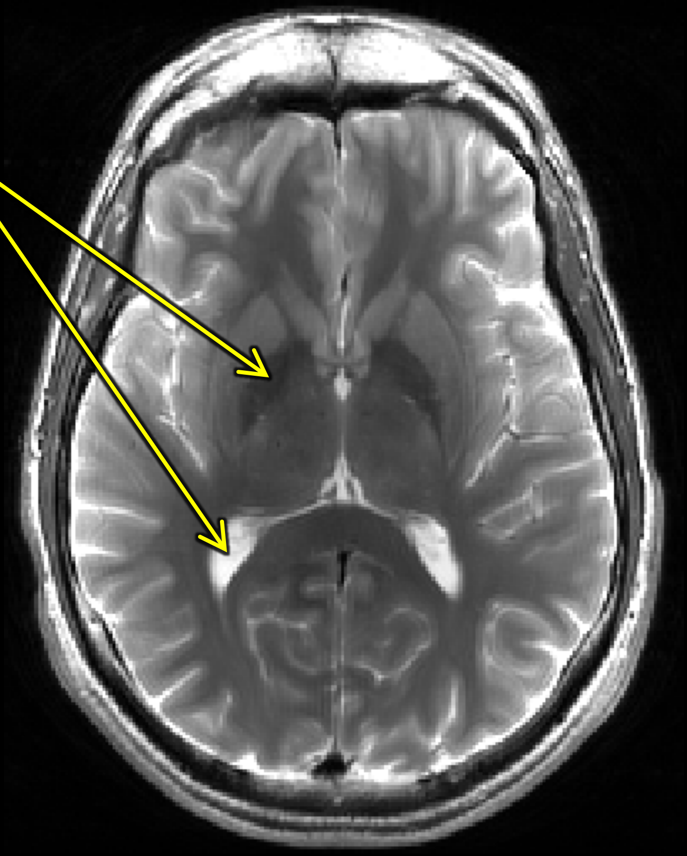


MRI is a physical measurement with spatial information

(1) Why are some regions bright, some dark?

IE: What are we physically measuring?

How do we make this measurement?



MRI is a physical measurement with spatial information

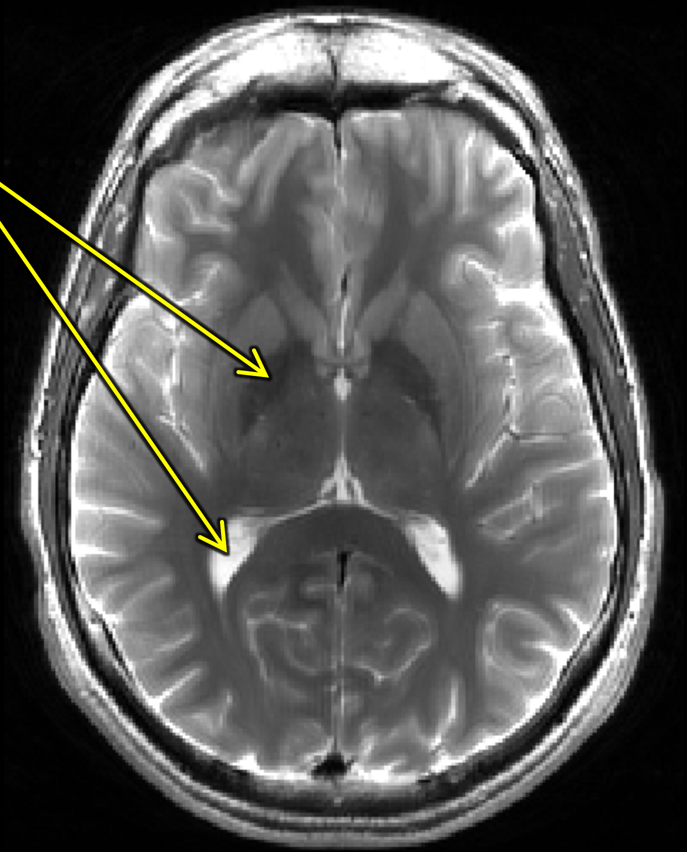
(1) Why are some regions bright, some dark?

IE: What are we physically measuring?

How do we make this measurement?

(2) How do we form an image?

IE: How do we get different mesurements from different spatial locations?

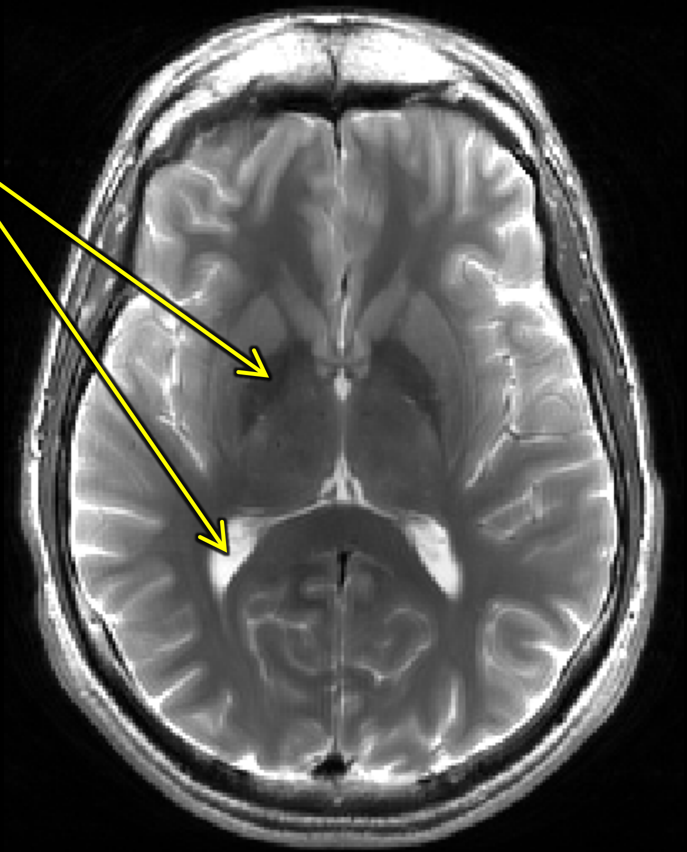


MRI is a physical measurement with spatial information

(1) Why are some regions bright, some dark?

IE: What are we physically measuring?

How do we make this measurement?

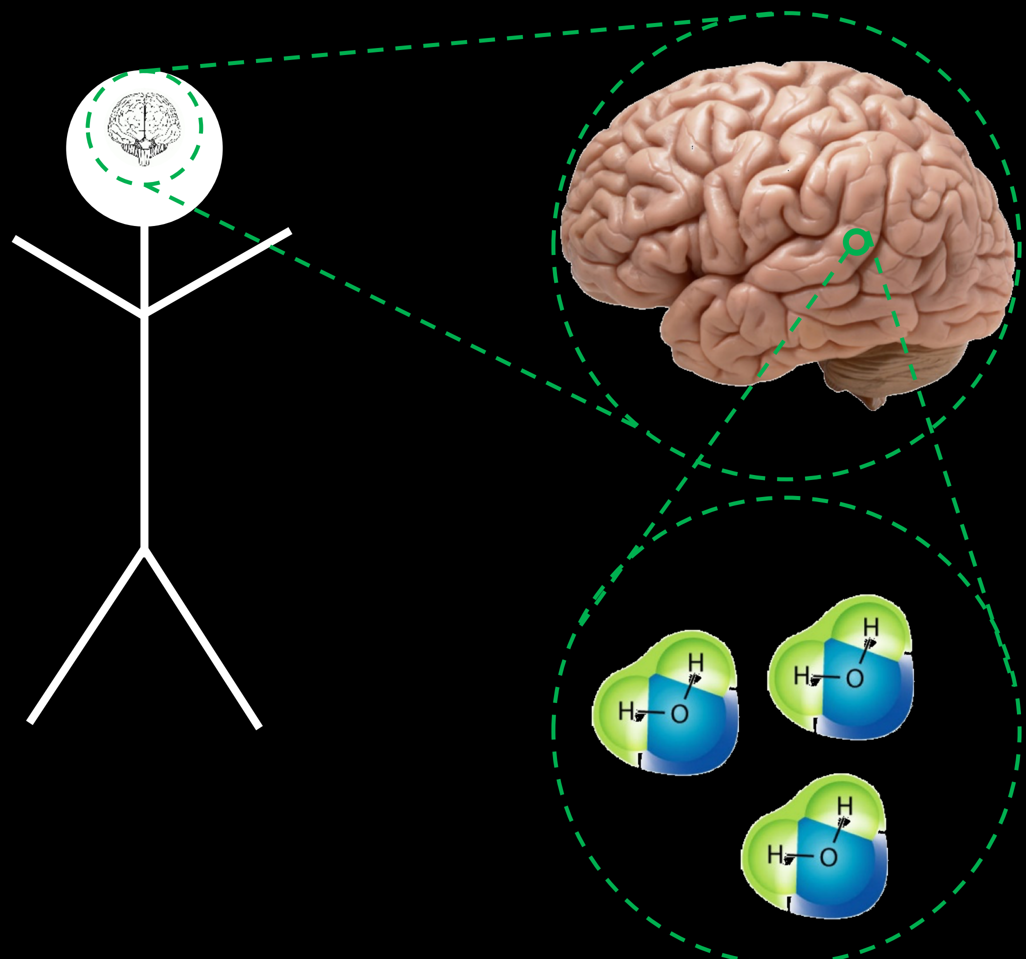


(2) How do we form an image?

IE: How do we get different mesurements from different spatial locations?

MRI Measures Water*

- *Most abundant substance in human body*



Your Brain:

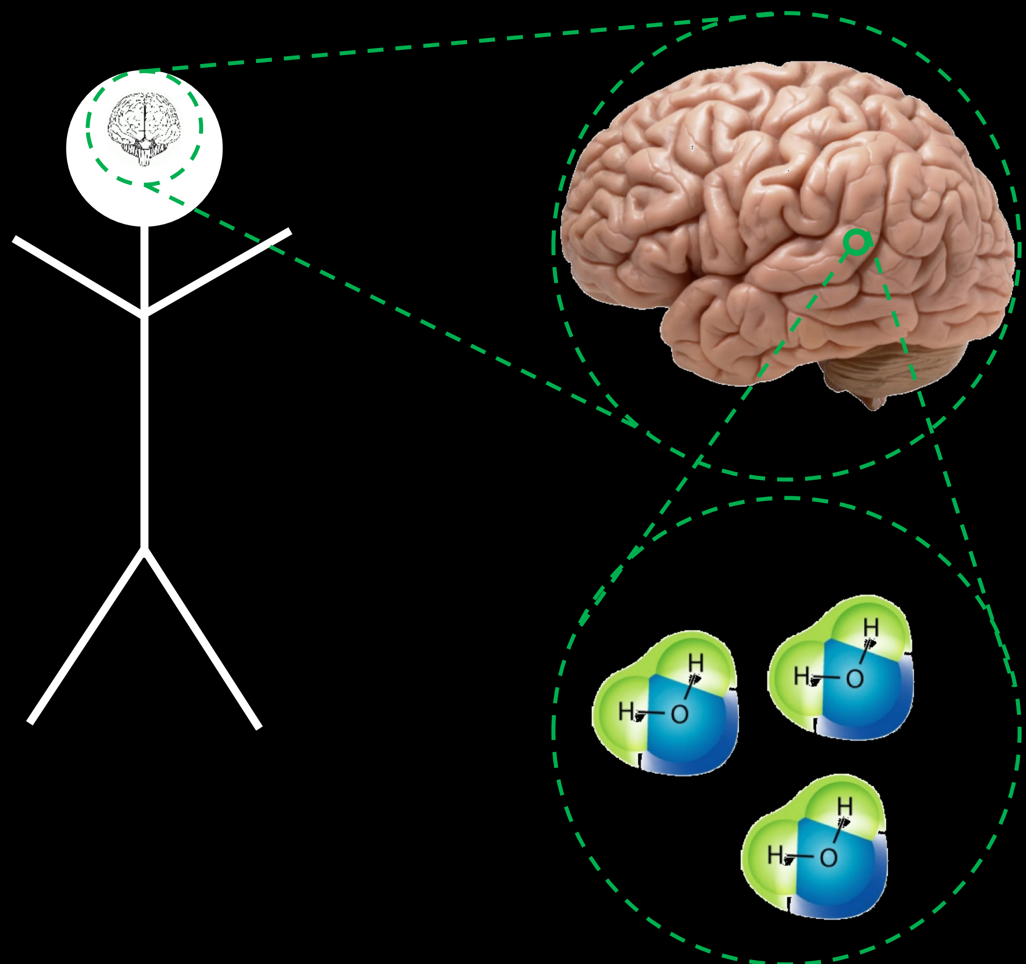
- Volume ≈ 1400 mL
- $\sim 5 \cdot 10^{25}$ x H_2O

1mm³ of Brain:

- $\sim 3 \cdot 10^{19}$ x H_2O

MRI Measures Water*

- *Most abundant substance in human body*
- *Provides a wide range of diagnostic information*



Your Brain:

- Volume $\approx$ 1400 mL
- $\sim 5 \cdot 10^{25}$ x H_2O

$1mm^3$ of Brain:

- $\sim 3 \cdot 10^{19}$ x H_2O

(N)MRI uses NMR to measure ^{1}H nuclei in water

(N)MR : (Nuclear) Magnetic Resonance

Measure Signal from
Atomic Nuclei (usually ^{1}H)

(N)MRI uses NMR to measure ^{1}H nuclei in water

(N)MR : (Nuclear) Magnetic Resonance

Measure Signal from Atomic Nuclei (usually ^{1}H)

Measure Magnetic Properties of ^{1}H

(N)MRI uses NMR to measure ^{1}H nuclei in water

(N)MR : (Nuclear) Magnetic Resonance

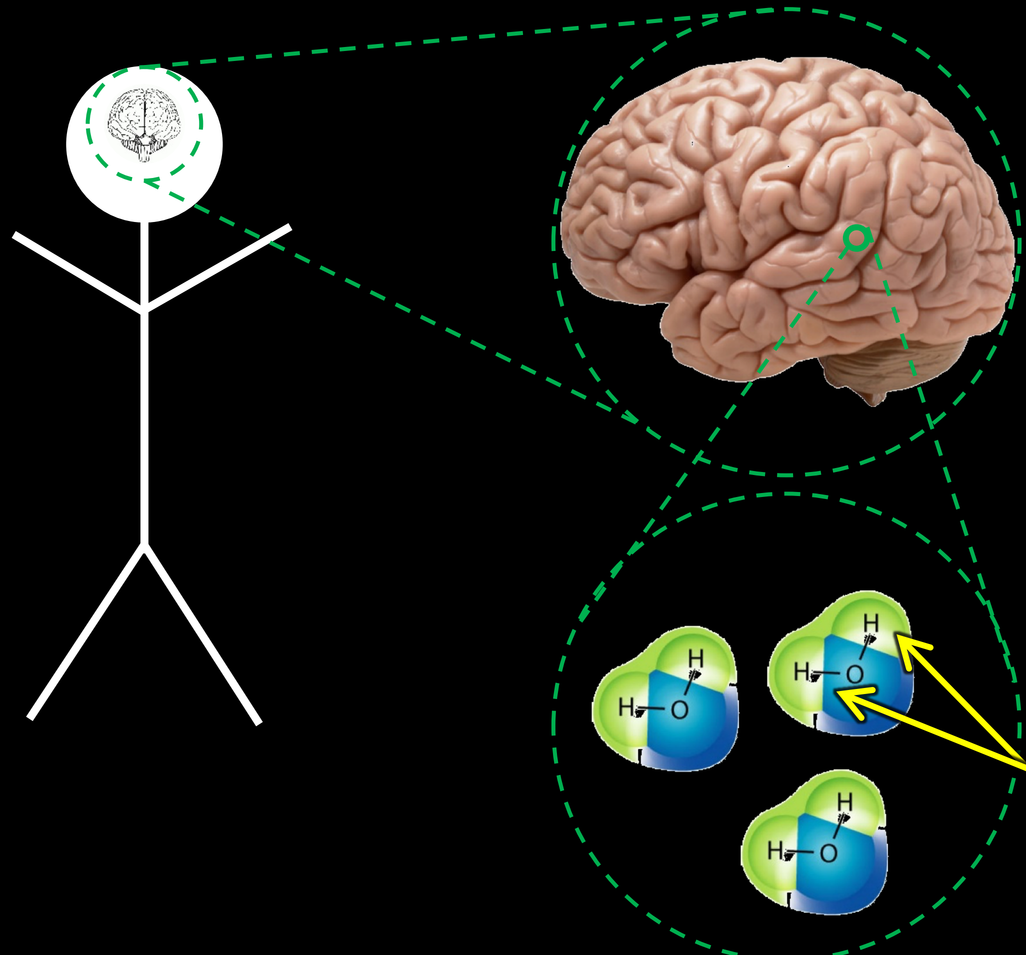
Measure Signal from Atomic Nuclei (usually ^{1}H)

Measure Magnetic Properties of ^{1}H

Make Measurement by exploiting *resonance* phenomenon (dependence on a specific frequency)

MRI uses NMR to measure ^{1}H nuclei in water

- *^{1}H gives strongest NMR signal among stable elements*



Your Brain:

- Volume $\approx$ 1400 mL
- $\sim 5 \cdot 10^{25}$ x H_2O
- $\mathbf{\sim 10^{26}}$ x ^{1}H nuclei

$1mm^3$ of Brain:

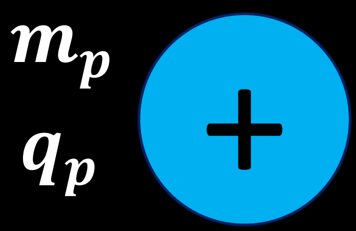
- $\sim 3 \cdot 10^{19}$ x H_2O
- $\mathbf{\sim 6 \cdot 10^{19}}$x ^{1}H nuclei

99.98% of Hydrogen is ^{1}H isotope

^{1}H Nucleus = Proton

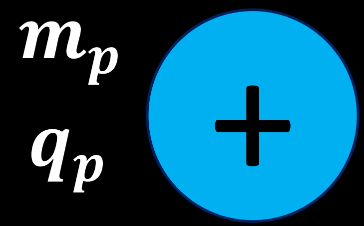
NMR measures proton magnetism

- Proton Mass: $\boldsymbol{m_p} = 1.7 \cdot 10^{-27}\ [kg]$
- Proton Charge: $\boldsymbol{q_p} = 1.6 \cdot 10^{-19}\ [C]$



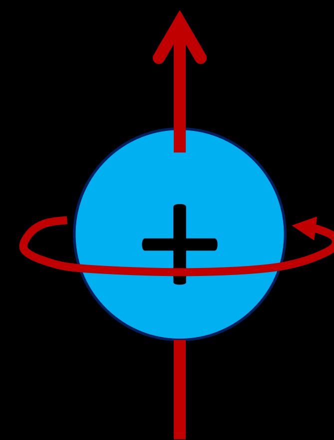
NMR measures proton magnetism

- Proton Mass: $\boldsymbol{m_p} = 1.7 \cdot 10^{-27}\ [kg]$
- Proton Charge: $\boldsymbol{q_p} = 1.6 \cdot 10^{-19}\ [C]$



Proton can be thought of as spinning*

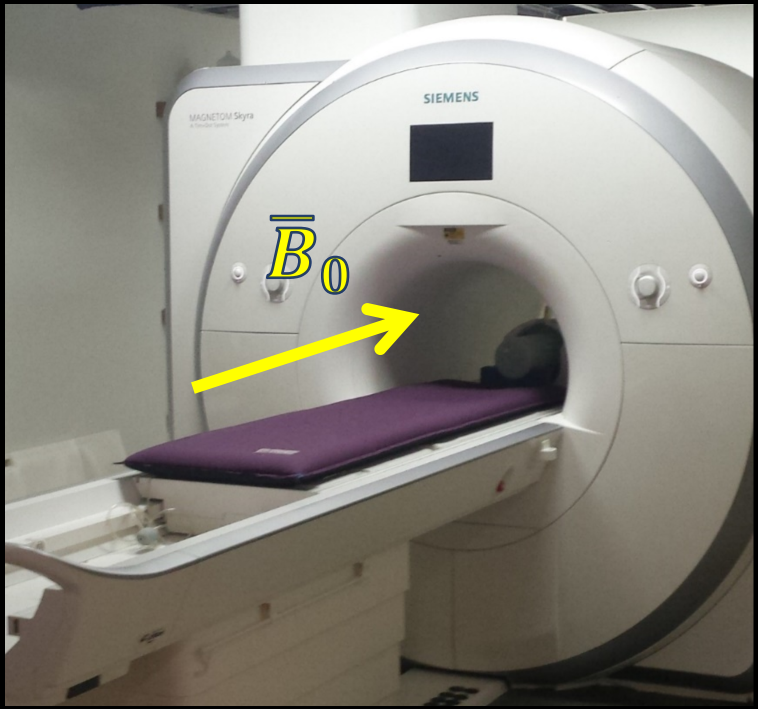
- "Spin" Angular Momentum: $\bar{\boldsymbol{S}}_p = \hbar \cdot \sqrt{\frac{3}{4}}$
- Magnetic Dipole Moment: $\bar{\mu}_p = 2\pi\gamma_p \cdot \bar{S}_p$



"Gyromagnetic Ratio" $\gamma_p = 42.58\ \left[\frac{MHz}{T}\right]$

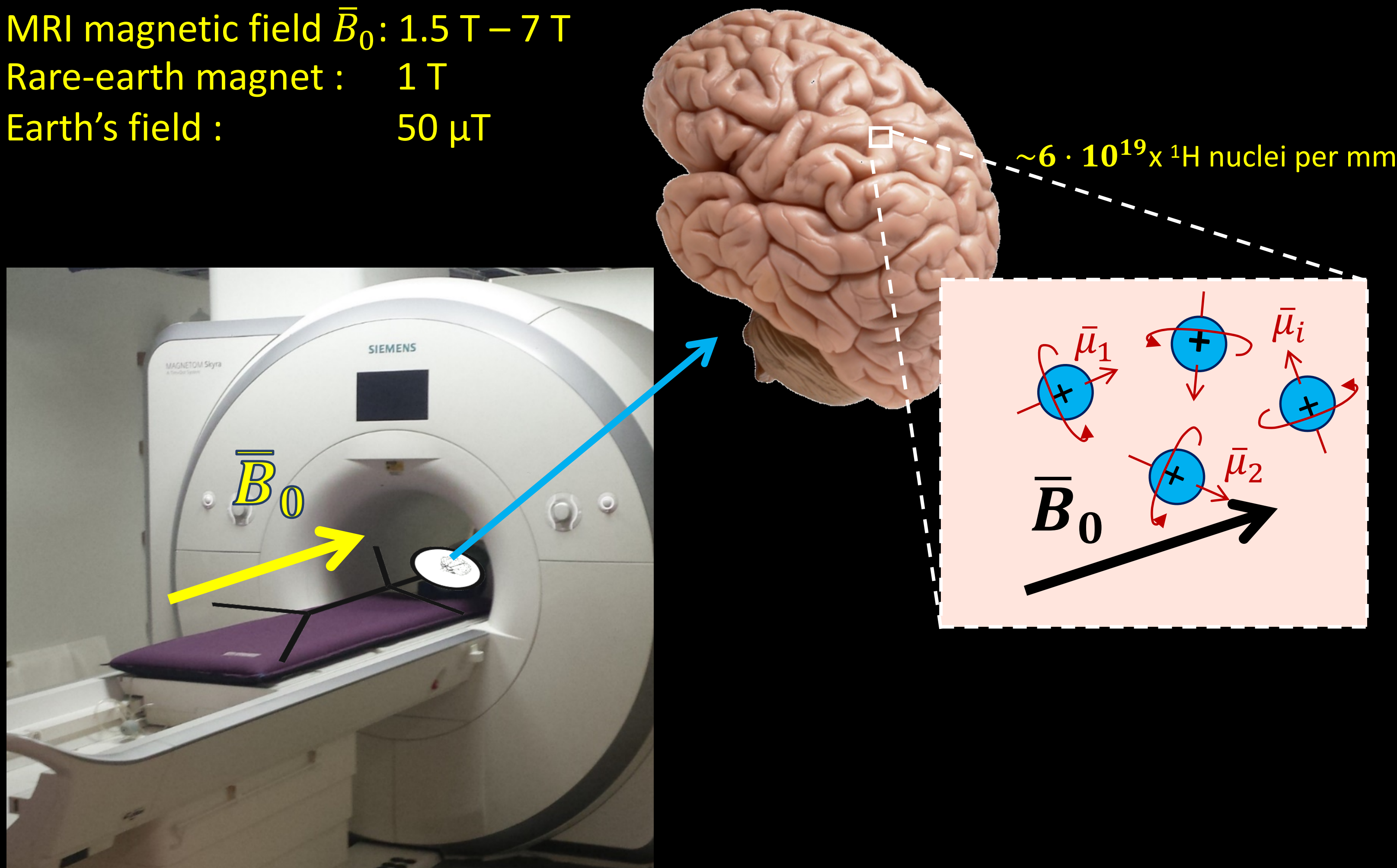
Protons align with MRI magnetic field

- Magnetic Fields
  - MRI magnetic field $\bar{B}_0$: 1.5 T – 7 T
  - Rare-earth magnet : 1 T
  - Earth's field : 50 µT



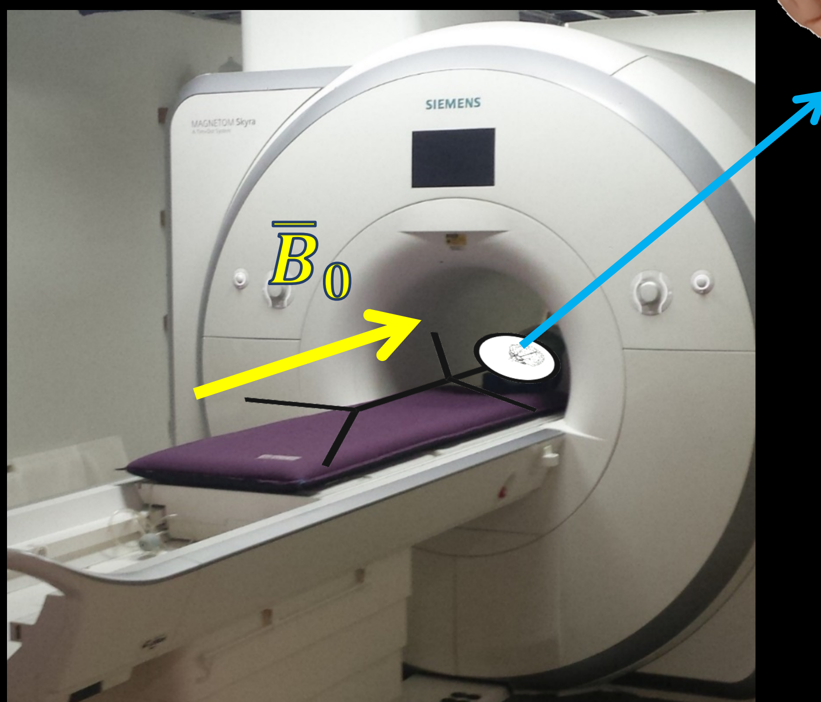
Protons align with MRI magnetic field

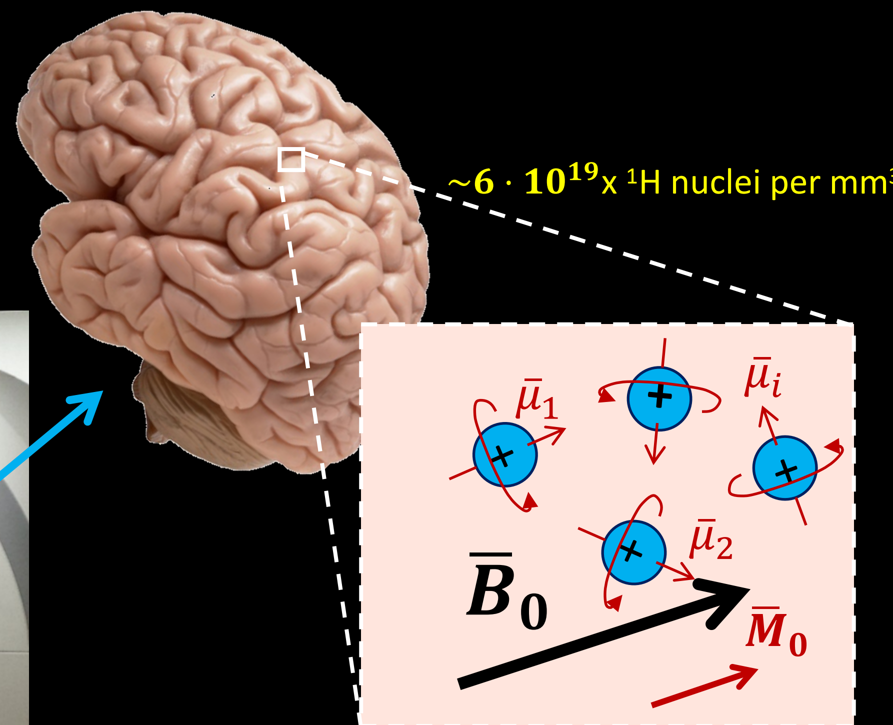
- Magnetic Fields
  - MRI magnetic field $\bar{B}_0$: 1.5 T – 7 T
  - Rare-earth magnet : 1 T
  - Earth's field : 50 µT



Protons align with MRI magnetic field

- Magnetic Fields
  - MRI magnetic field $\bar{B}_0$: 1.5 T – 7 T
  - Rare-earth magnet : 1 T
  - Earth's field : 50 µT
- Weak alignment due to random thermal fluctuations



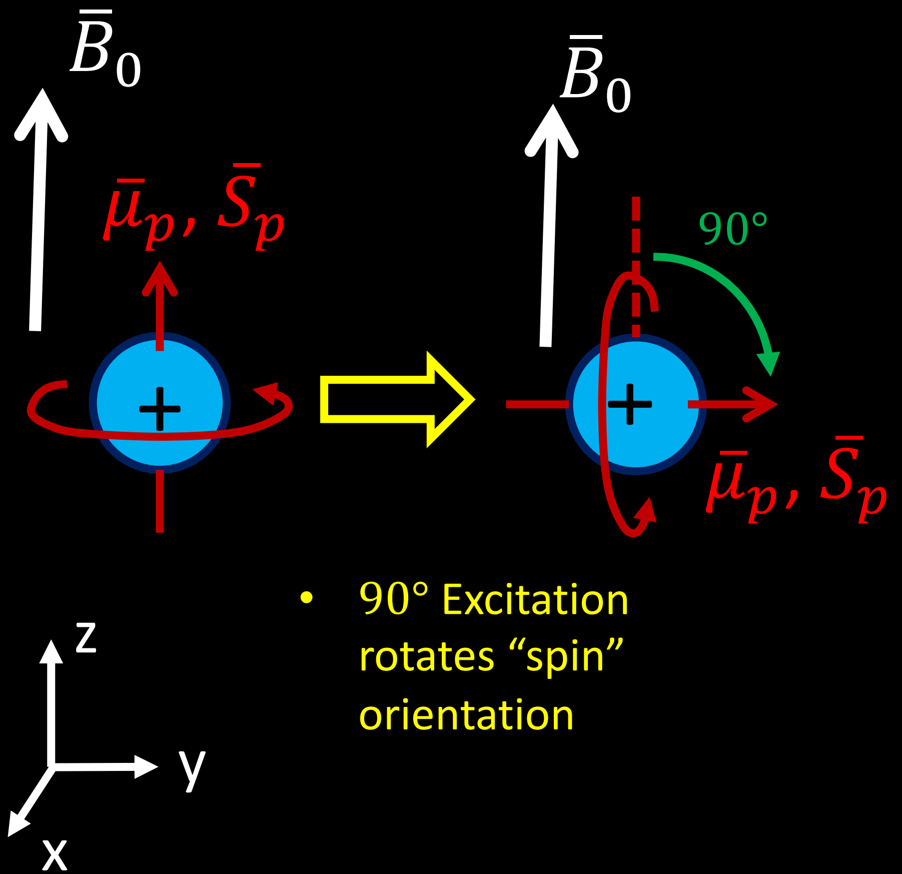


$$\bar{M}_0 = \sum \overline{\mu_i} \sim \frac{\gamma^2 h^2}{4 k_b T} \cdot \bar{B}_0$$

$\sim 10^{-5}$ of maximum available magnetization

NMR measurements involve "excitation" and "detection"

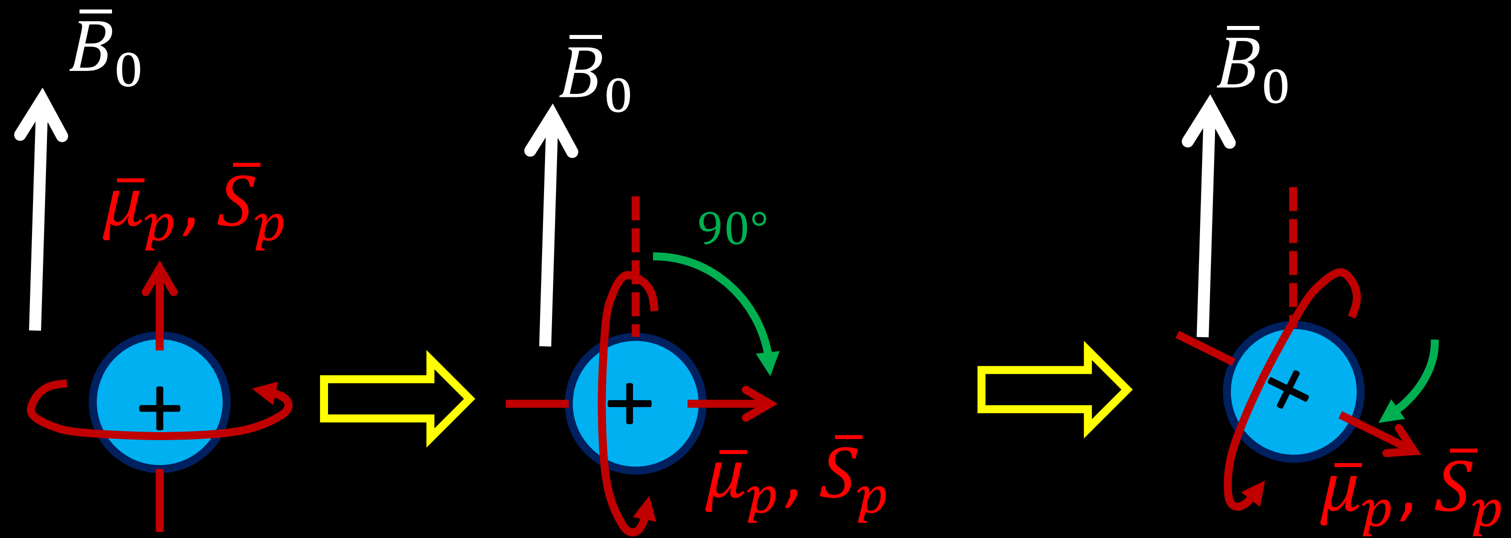
Excitation



- 90° Excitation rotates "spin" orientation

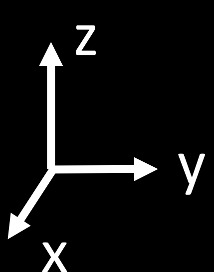
NMR measurements involve "excitation" and "detection"

Excitation | *Detection*



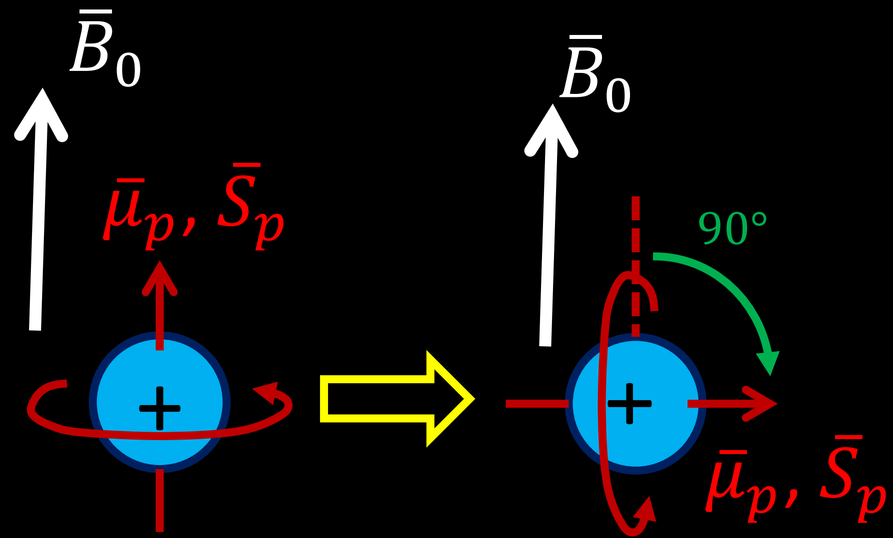
- 90° Excitation rotates "spin" orientation

- Proton precesses around $\bar{B}_0$
- Acquire signal



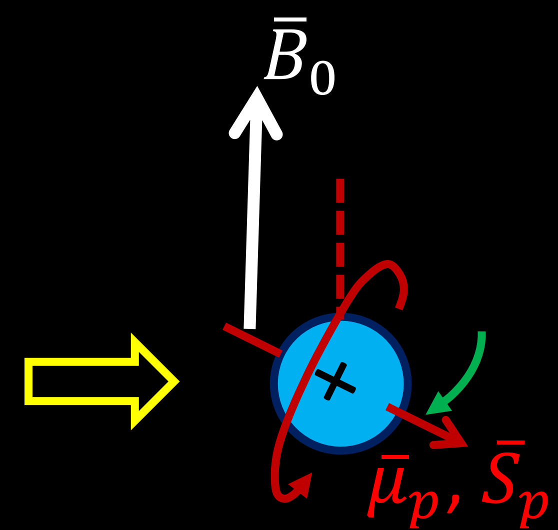
NMR measurements involve "excitation" and "detection"

Excitation

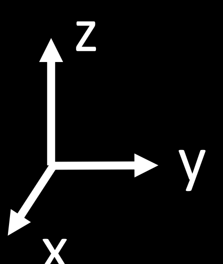


- 90° Excitation rotates "spin" orientation

Detection

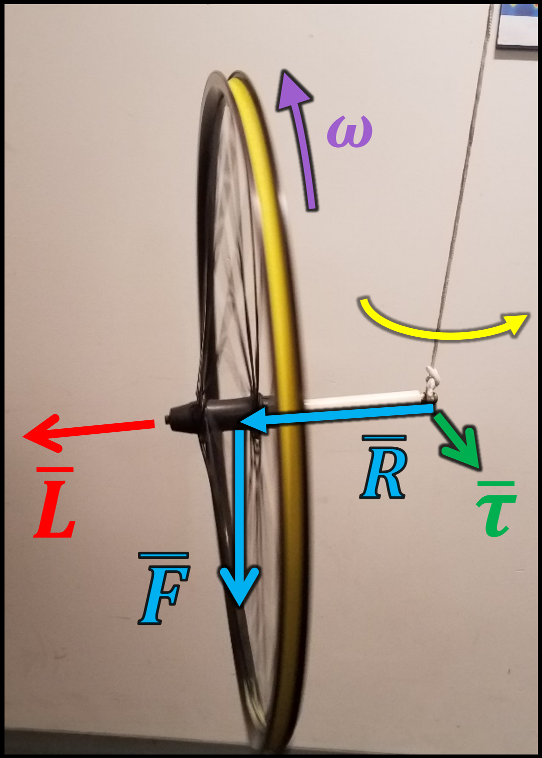


- Proton precesses around $\bar{B}_0$
- Acquire signal



A proton in a magnetic field precesses like a gyroscope

Gyroscope



Wheel Spinning

- Angular velocity ω

Angular momentum

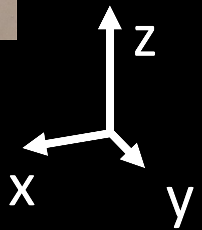
- $\overline{L} = I \cdot \omega \cdot \hat{R}$

Gravitational Force

- $\overline{F} = M \cdot g \cdot (-\hat{z})$

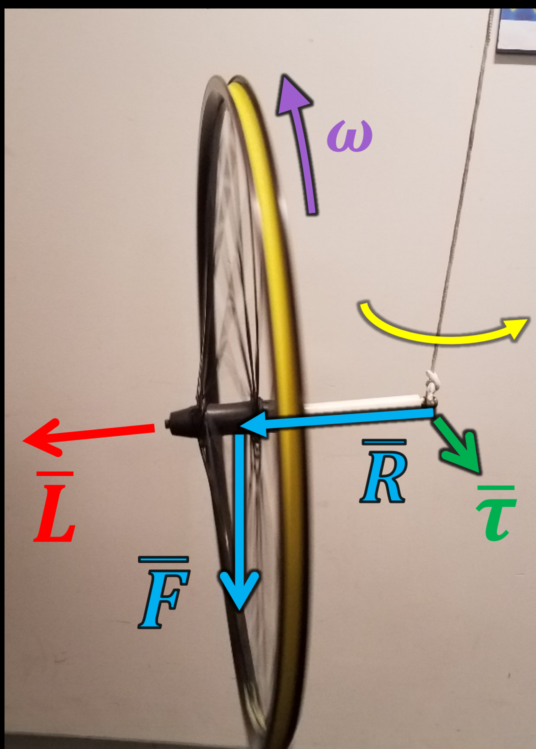
Torque on wheel

- $\overline{\tau} = \overline{R} \times \overline{F}$



A proton in a magnetic field precesses like a gyroscope

Gyroscope



Wheel Spinning

- Angular velocity ω

Angular momentum

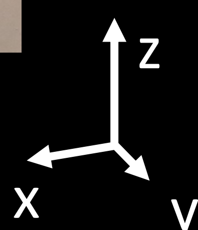
- $\bar{L} = I \cdot \omega \cdot \hat{R}$

Gravitational Force

- $\bar{F} = M \cdot g \cdot (-\hat{z})$

Torque on wheel

- $\bar{\tau} = \bar{R} \times \bar{F}$



Solve equation of motion:

$$\frac{d}{dt}\bar{L} = \bar{\tau}$$

$$\frac{d}{dt}\hat{R} = \frac{-MgR}{I\omega} \cdot (\hat{R} \times \hat{z})$$

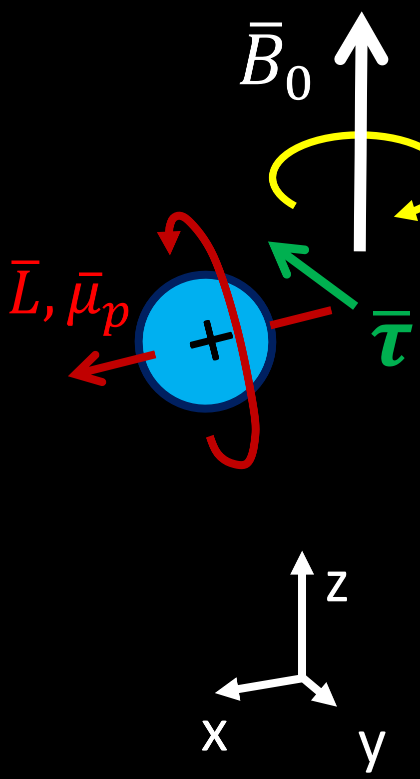
$$\hat{R}(t) = \cos(\Omega t)\,\hat{x} + \sin(\Omega t)\,\hat{y}$$

Precession with frequency Ω

- $\Omega = \frac{M \cdot g \cdot R}{I \cdot \omega}$

A proton in a magnetic field precesses like a gyroscope

Proton in Magnetic Field



Angular momentum

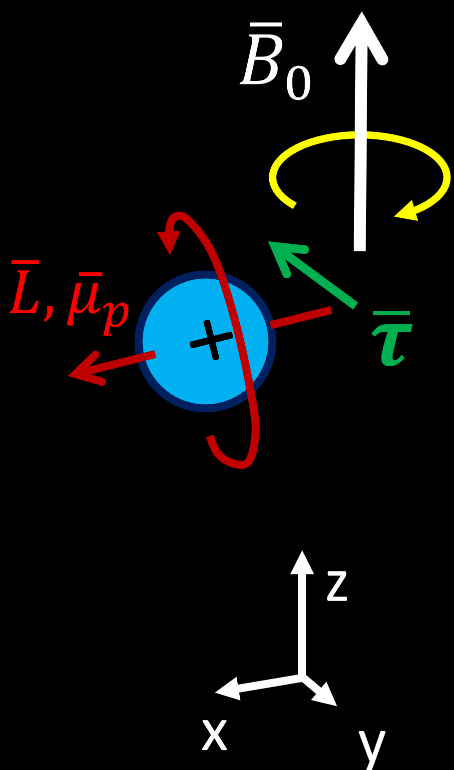
- $\bar{L} = \bar{S}_p$
- $\bar{\mu}_p = \gamma \cdot \bar{S}_p$

Torque on proton

- $\bar{\tau} = \bar{\mu}_p \times \bar{B}_0$

A proton in a magnetic field precesses like a gyroscope

Proton in Magnetic Field



Angular momentum

- $\bar{L} = \bar{S}_p$
- $\bar{\mu}_p = \gamma \cdot \bar{S}_p$

Torque on proton

- $\bar{\tau} = \bar{\mu}_p \times \bar{B}_0$

Solve equation of motion:

$$\frac{d}{dt}\bar{L} = \bar{\tau}$$

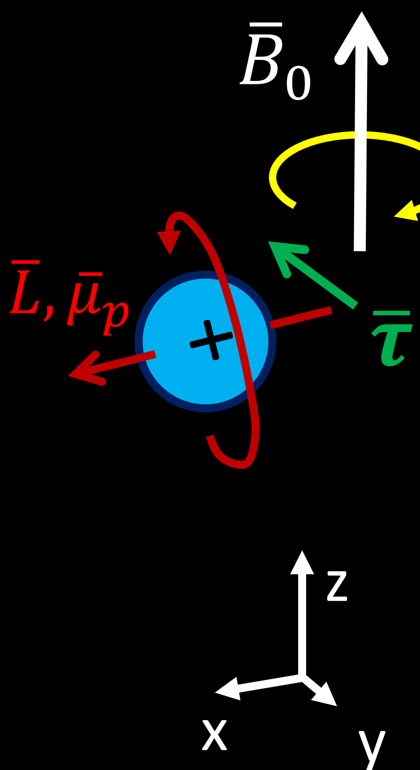
"Bloch Equation"

$$\frac{d}{dt}\bar{\mu}_p = \gamma \cdot (\bar{\mu}_p \times \bar{B})$$

$$\frac{d}{dt}\bar{\mu}_p = \gamma \cdot B_0 \cdot (\bar{\mu}_p \times \hat{z})$$

A proton in a magnetic field precesses like a gyroscope

Proton in Magnetic Field



Angular momentum
- $\bar{L} = \bar{S}_p$
- $\bar{\mu}_p = \gamma \cdot \bar{S}_p$

Torque on proton
- $\bar{\tau} = \bar{\mu}_p \times \bar{B}_0$

Solve equation of motion:

$$\frac{d}{dt}\bar{L} = \bar{\tau}$$

"Bloch Equation" $$\frac{d}{dt}\bar{\mu}_p = \gamma \cdot (\bar{\mu}_p \times \bar{B})$$

$$\frac{d}{dt}\bar{\mu}_p = \gamma \cdot B_0 \cdot (\bar{\mu}_p \times \hat{z})$$

$$\bar{\mu}_p(t) = \mu_p \cdot (\cos(\Omega t)\,\hat{x} - \sin(\Omega t)\,\hat{y})$$

Precession with frequency Ω
- $\Omega = 2\pi\gamma \cdot B_0 \equiv 2\pi f_L$
- f_L : Larmor Frequency

Remember This Equation!

$$f_L = \gamma \cdot B$$

Magnetic Field Strength

- *In Precession :* $B = B_0$

Gyromagnetic Ratio (Constant)

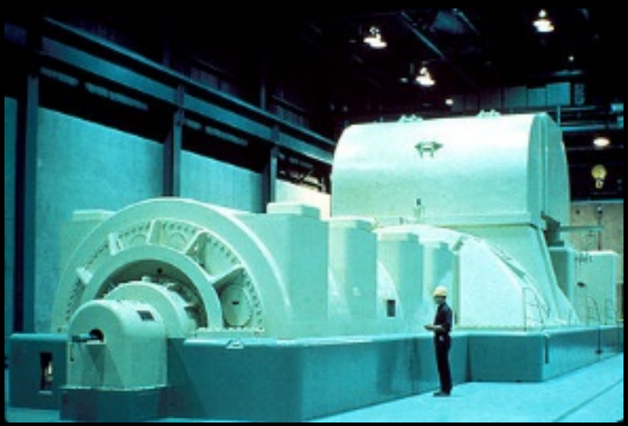
- $\gamma = 42.576 \cdot 10^6$ *MHz / T*

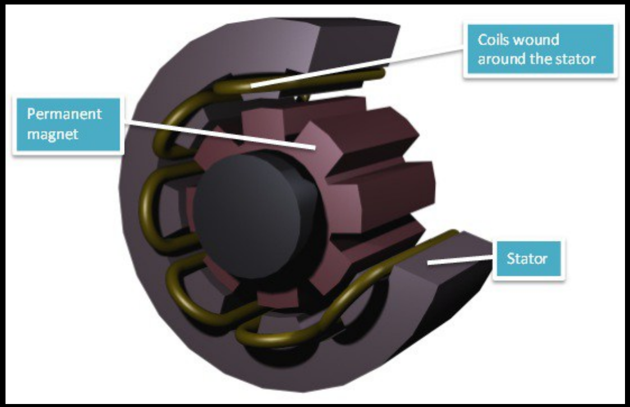
Larmor Frequency

- Proton precession frequency

Rotating magnets generate voltage in an electrical coil

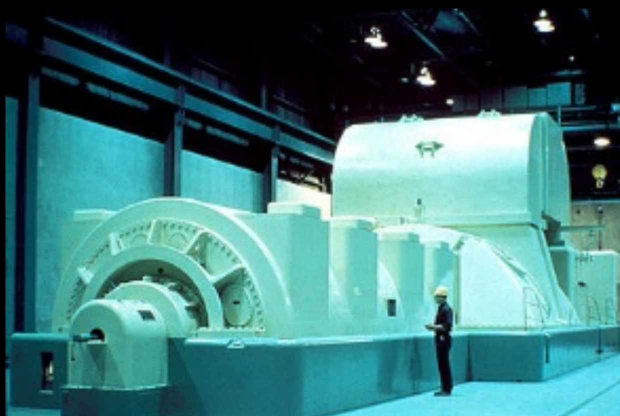
Large magnets: power generation

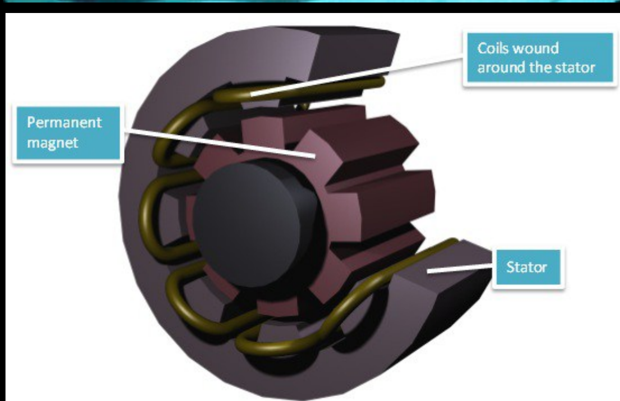




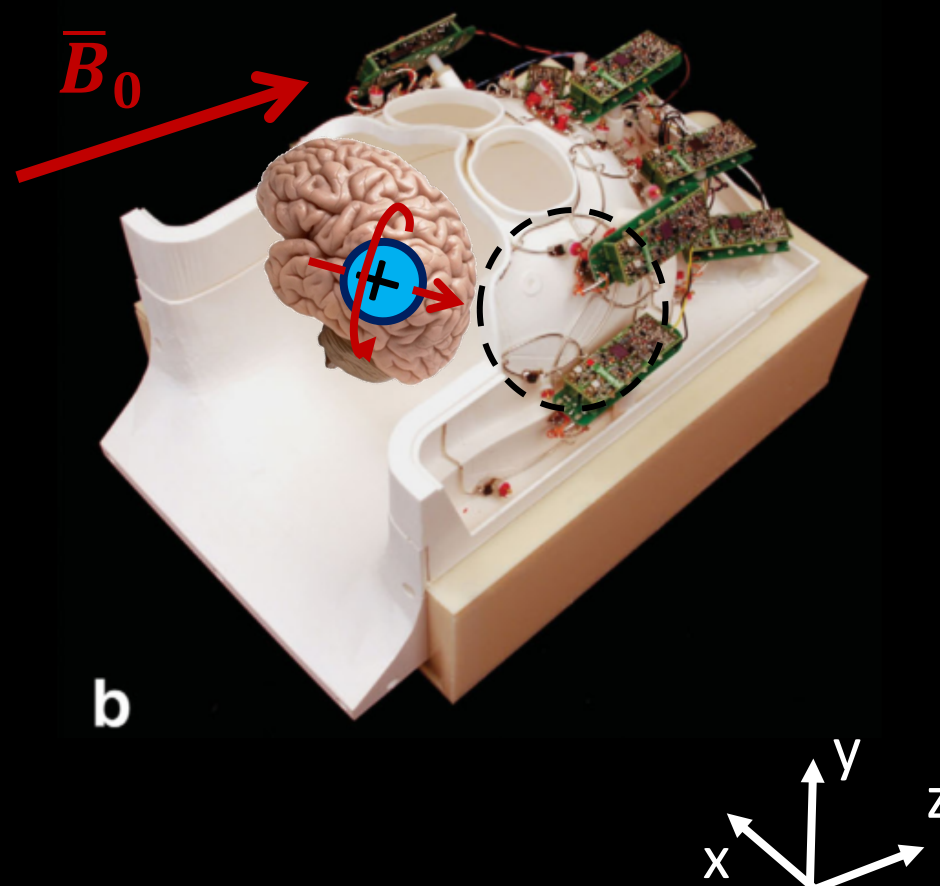
Rotating magnets generate voltage in an electrical coil

Large magnets: power generation

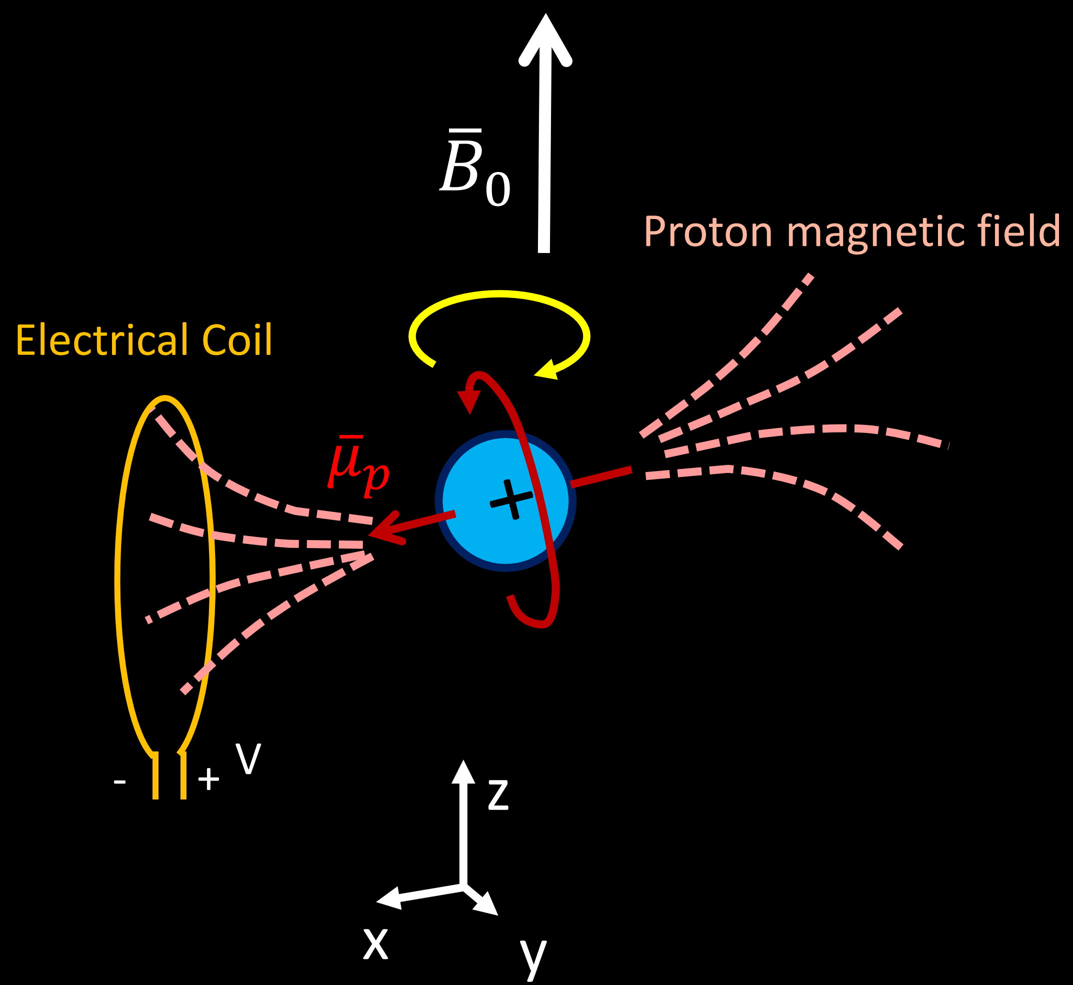




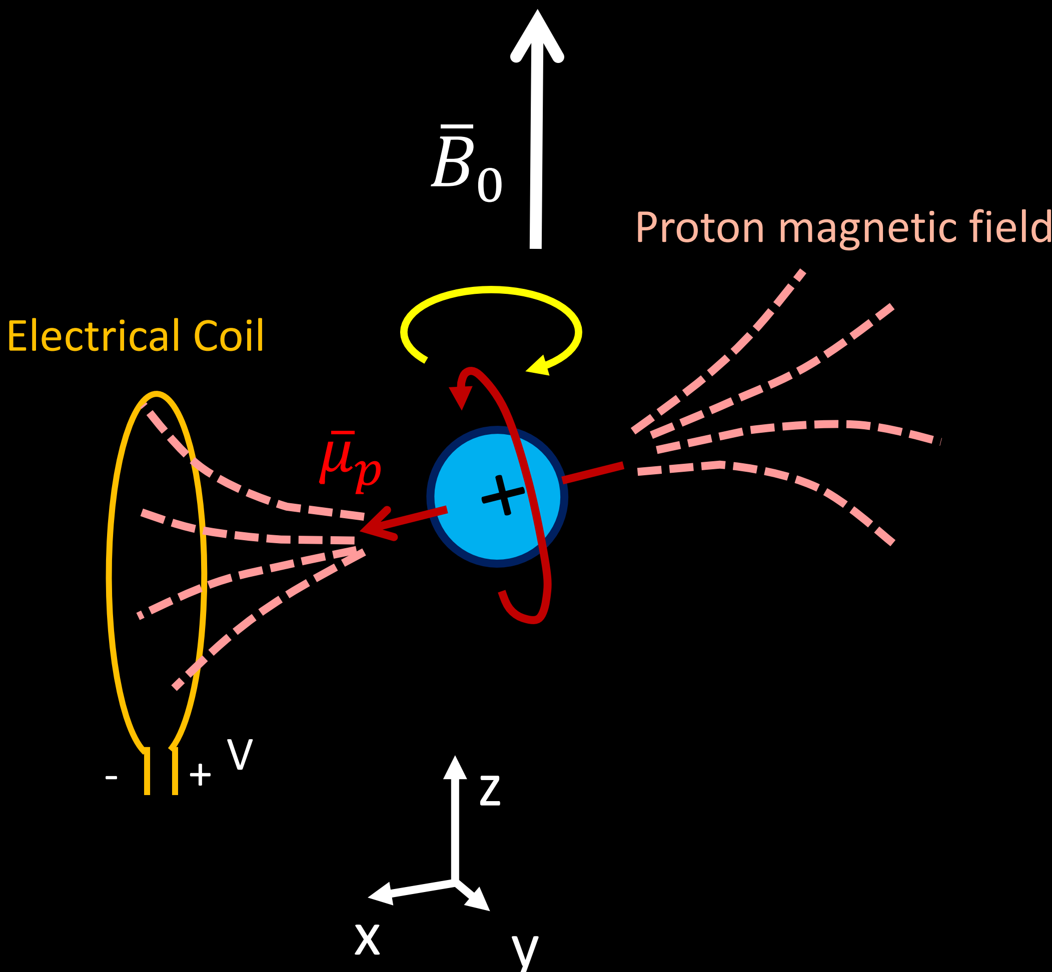
Tiny magnets: Proton NMR detection



Rotating magnets generate voltage in an electrical coil



Rotating magnets generate voltage in an electrical coil



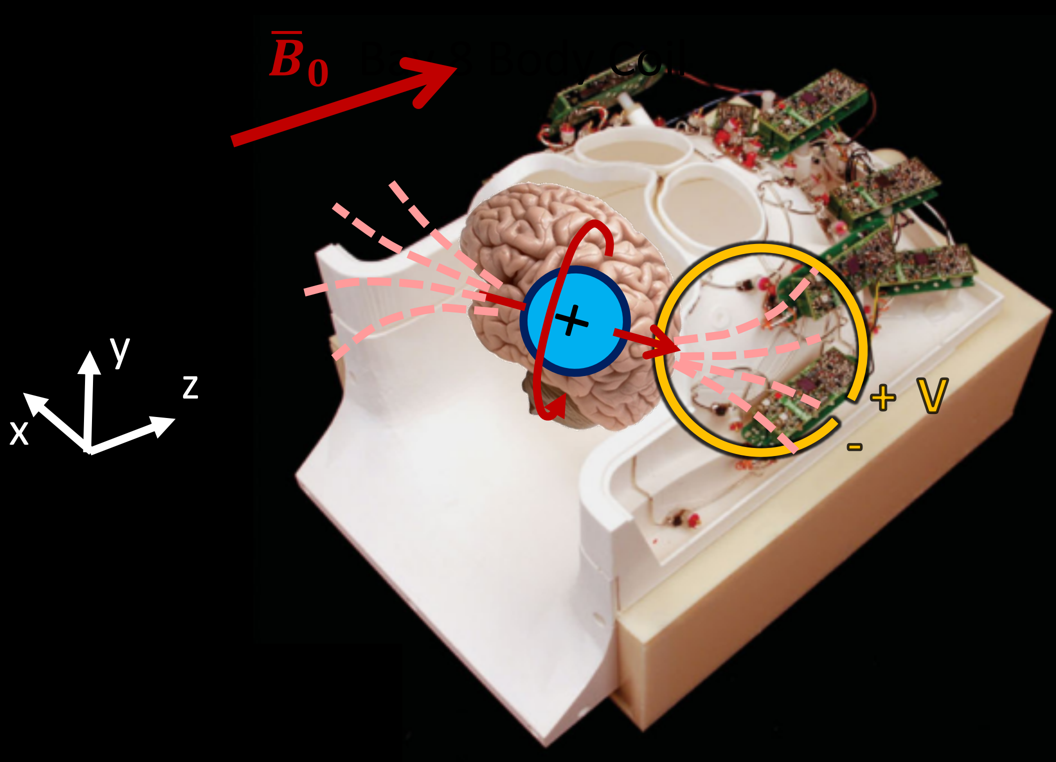
Faraday's Law

$$-\frac{d}{dt}\Phi_B = V$$

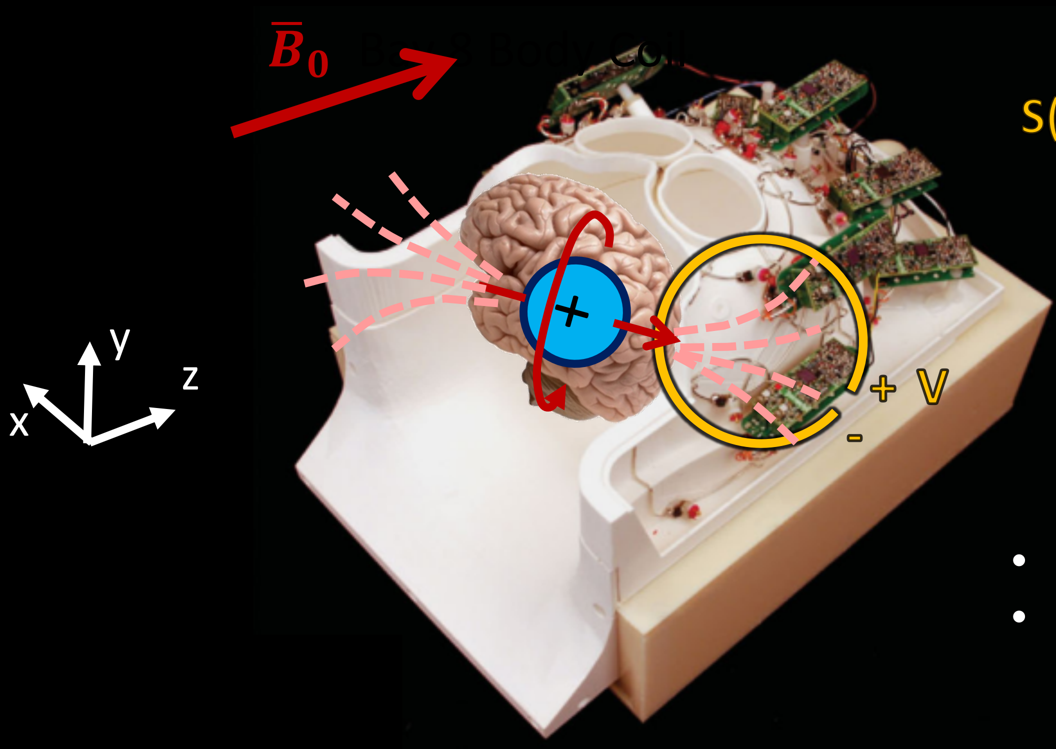
Changing magnetic flux through a loop

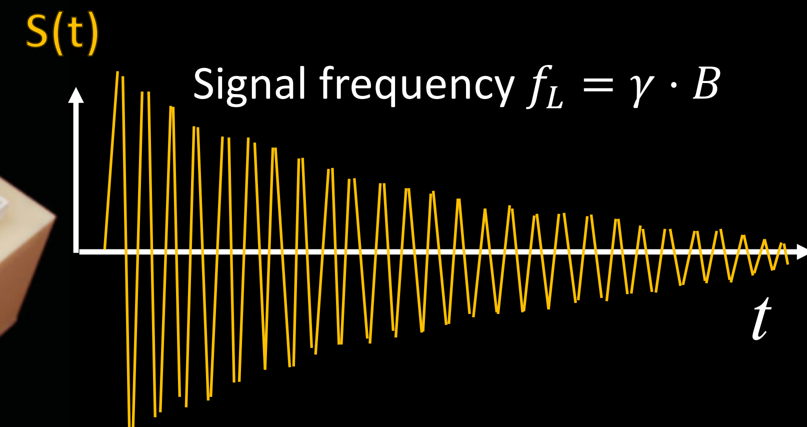
Voltage across loop

Electronic circuits sample coil voltage to acquire data



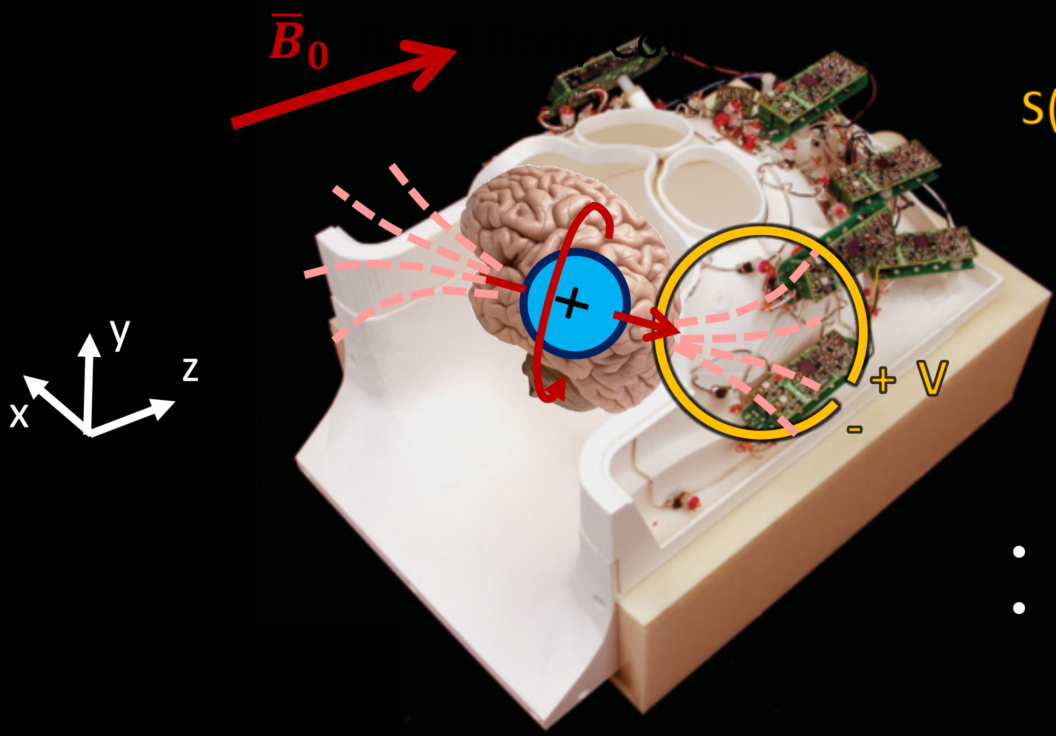
Electronic circuits sample coil voltage to acquire data

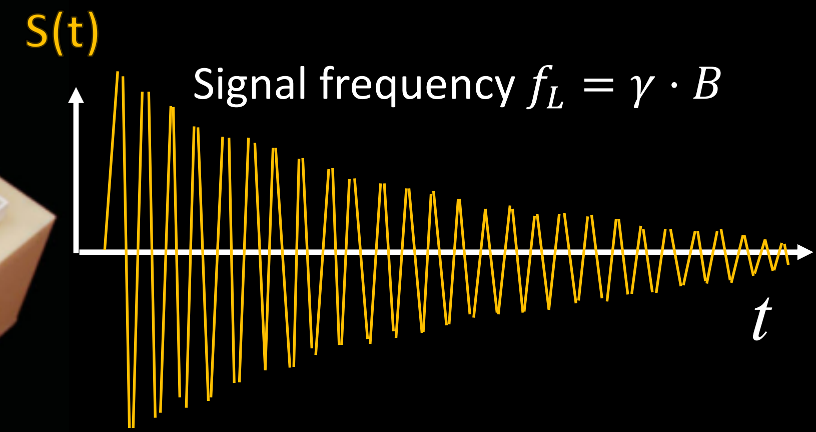




- Radio-Frequency (RF) signal
- Example frequency values:
  - $f = 64\ MHz$ at 1.5 T
  - $f = 123.2 MHz$ at 3 T
  - $f = 300\ MHz$ at 7 T
- No meaningful units

Electronic circuits sample coil voltage to acquire data

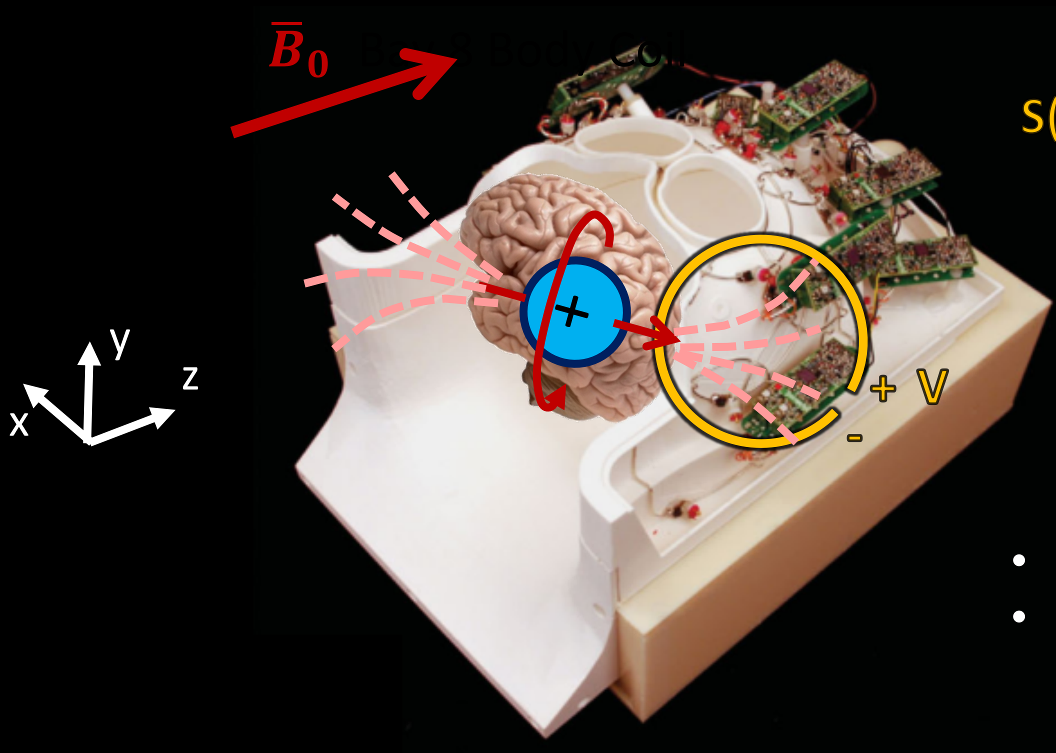


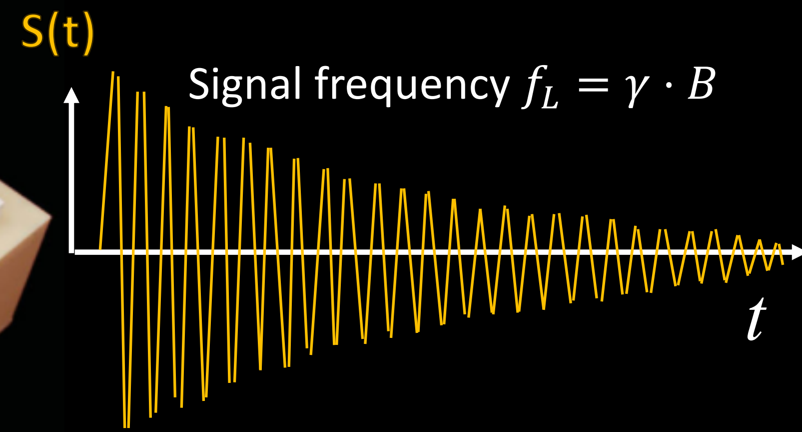


- Radio-Frequency (RF) signal
- Example frequency values:
  - $f = 64\ MHz$ at 1.5 T
  - $f = 123.2 MHz$ at 3 T
  - $f = 300\ MHz$ at 7 T
- No meaningful units

Protons → Voltage → Data

Electronic circuits sample coil voltage to acquire data

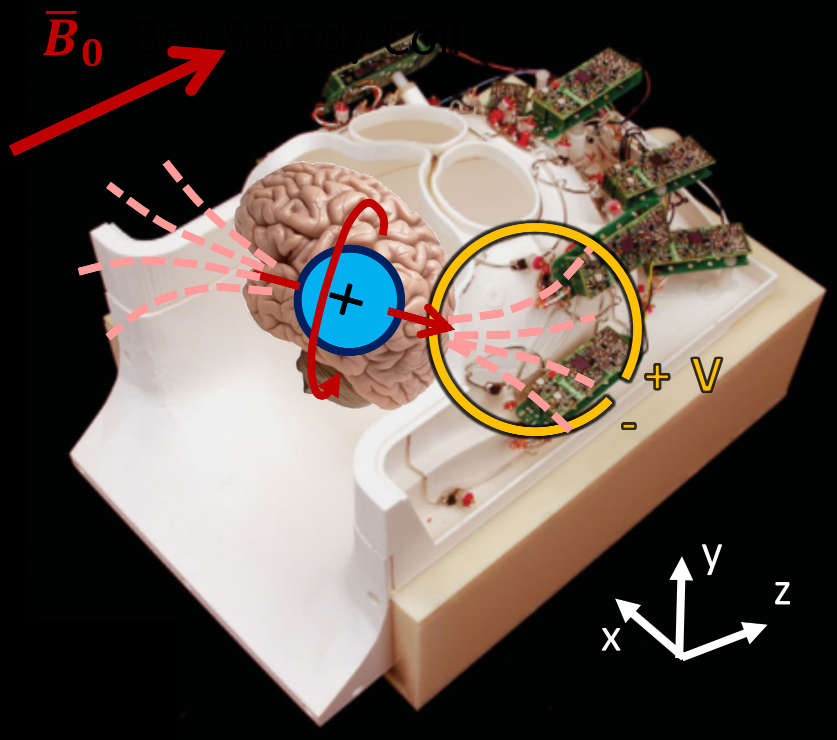


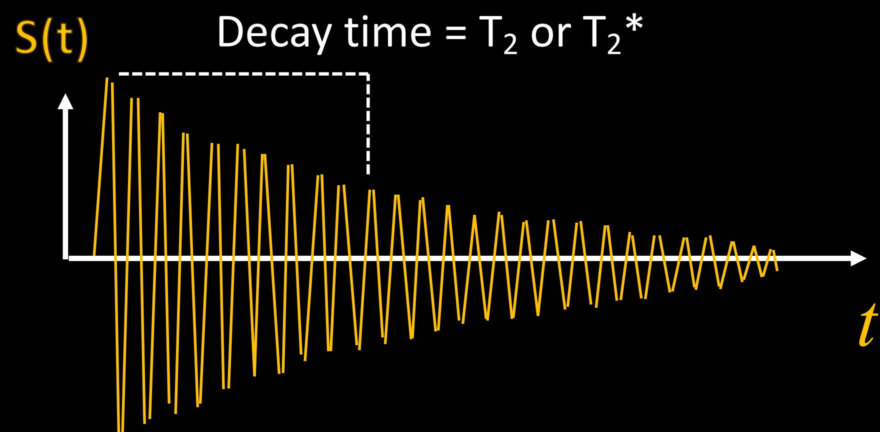


- Radio-Frequency (RF) signal
- Example frequency values:
  - $f = 64\ MHz$ at 1.5 T
  - $f = 123.2 MHz$ at 3 T
  - $f = 300\ MHz$ at 7 T
- No meaningful units

Protons → Voltage → Data → Deep learning

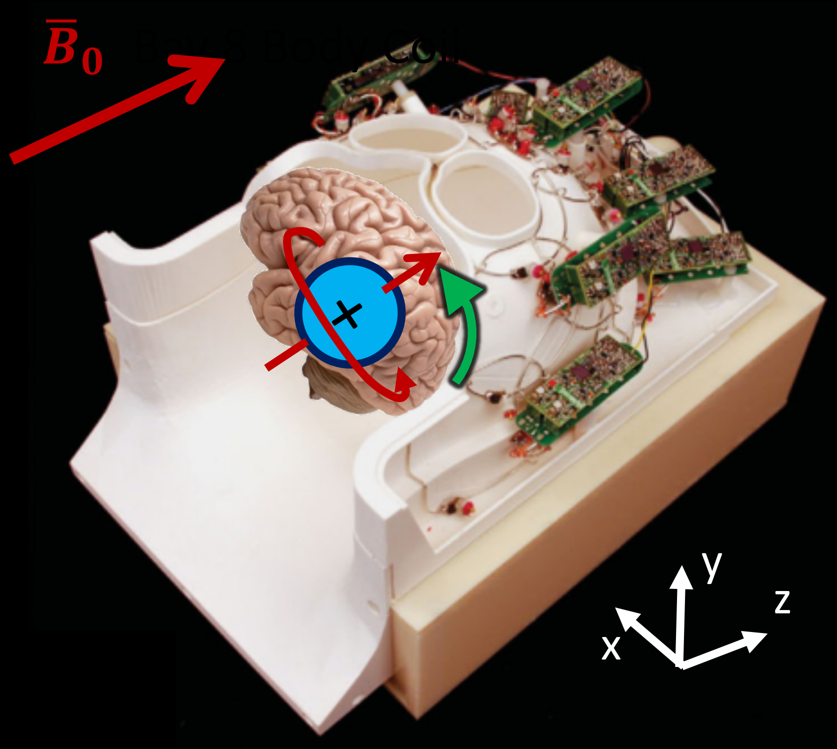
The NMR signal decays with time constant T_2 or T_2*



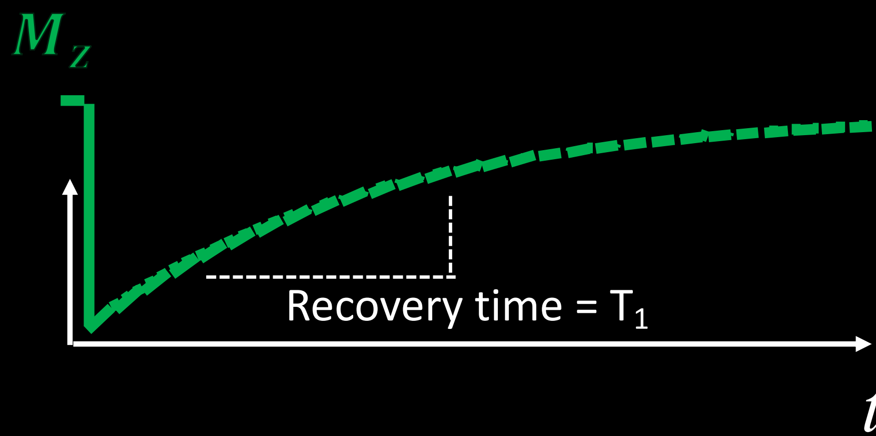


- $T_2^* < T_2$ due to inhomogeneous magnetic field
- Typical values – T_2
    - CSF ~ 1 s
    - Gray matter/white matter/blood ~ 100ms

Magnetization recovers with time constant T1



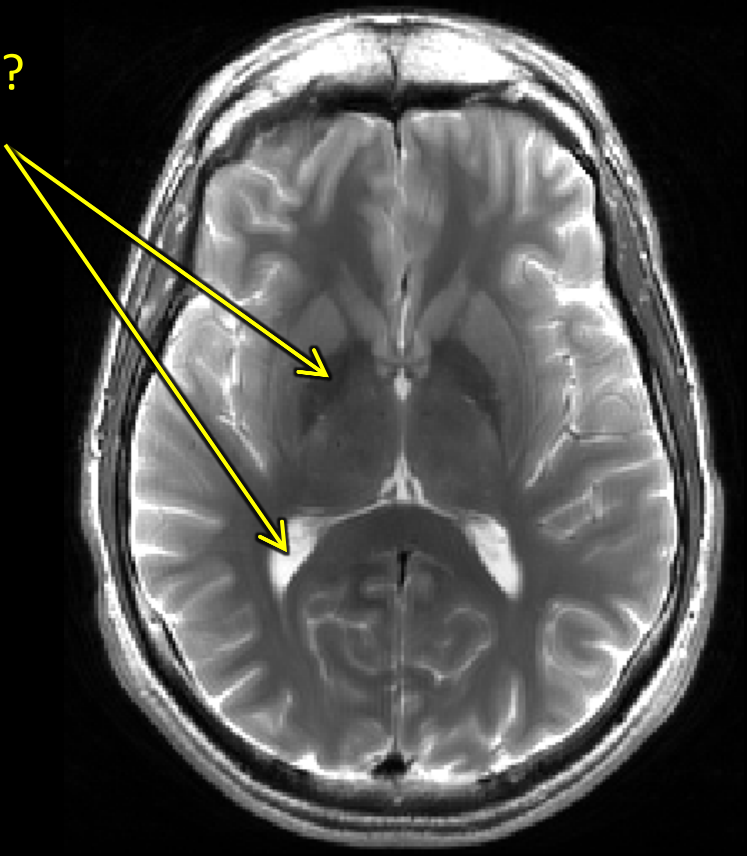
$\bar{\mu}_p$ points along $\bar{B}_0$ again



- $T_1 > T_2$
- Typical values – T_1
  - CSF ~ 3.5 s
  - Gray matter/white matter/blood ~ 1 s

MRI is a physical measurement with spatial information

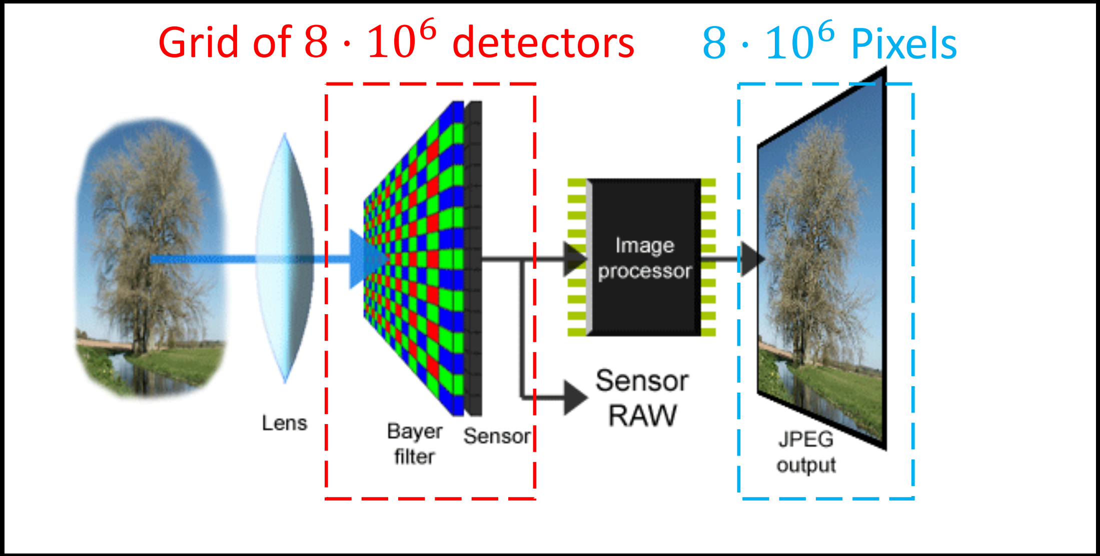
(1) Why are some regions bright, some dark?
IE: What are we physically measuring?
How do we make this measurement?



(2) How do we form an image?
IE: How do we get different mesurements from different spatial locations?

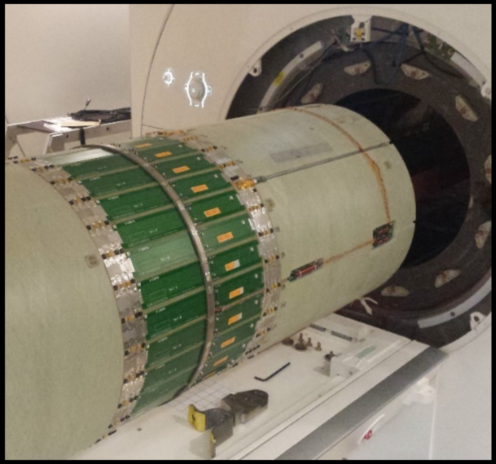
MR Imaging is unlike most imaging

Digital Camera:

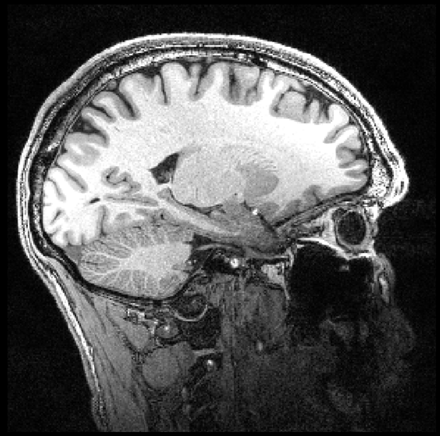


MRI:

1 NMR detector coil



$8 \cdot 10^6$ *Voxels*

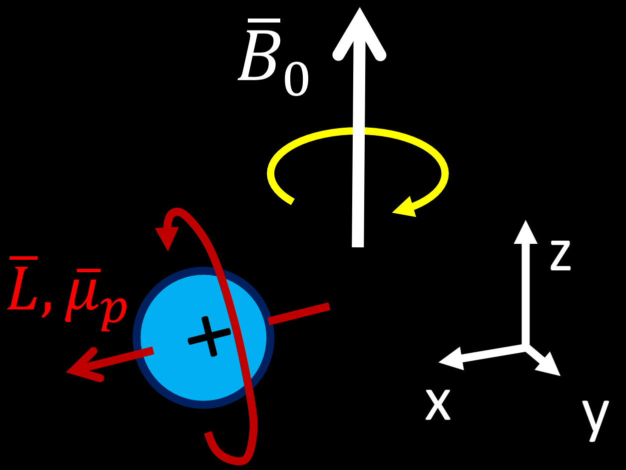


Preliminaries to Imaging

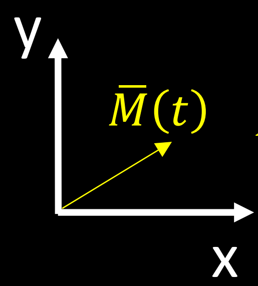
- The MRI signal is a complex number
- The measured signal is the sum of the signals from every location in the sample
- MRI data is acquired over a period of time

The MRI signal is a complex number

Physical Picture



2D Vector Picture

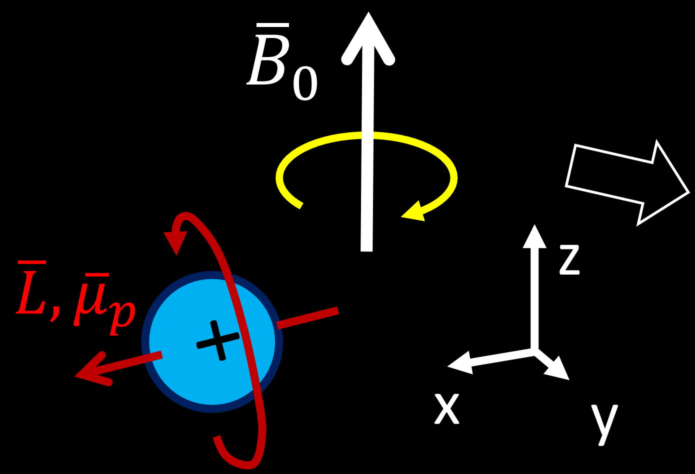


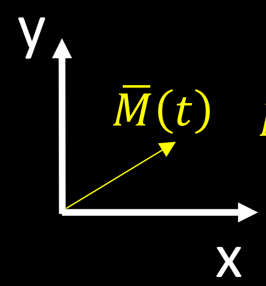
$$\bar{M}(t) = M_0 \cdot (\cos(2\pi f_L t)\,\hat{x} - \sin(2\pi f_L t)\,\hat{y})$$

The MRI signal is a complex number

2D Vector Picture

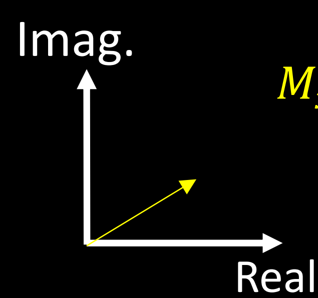
Physical Picture





$$\bar{M}(t) = M_0 \cdot (\cos(2\pi f_L t)\,\hat{x} - \sin(2\pi f_L t)\,\hat{y})$$

Complex Number Picture

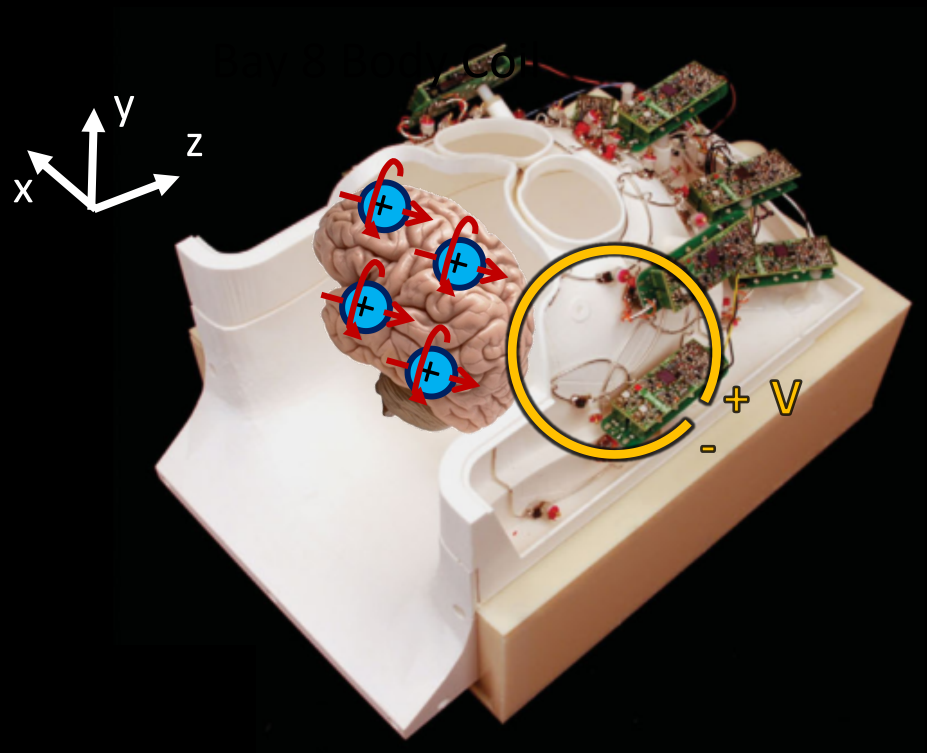


$$M_{xy}(t) = M_0 \cdot (\cos(2\pi f_L t) - \mathrm{j} \cdot \sin(2\pi f_L t))$$

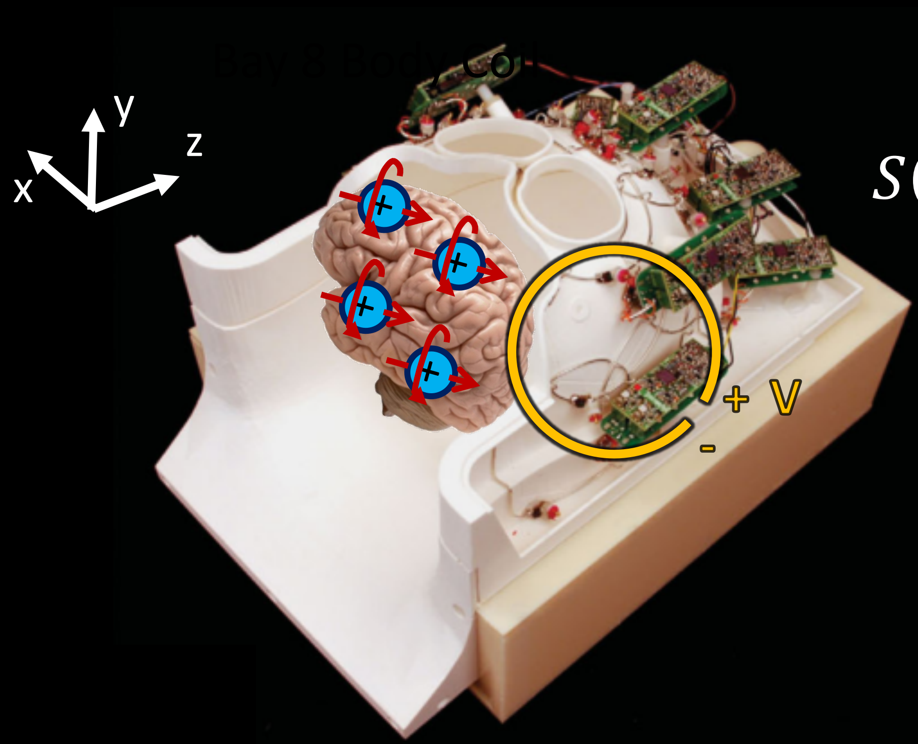
$$= M_0 \cdot e^{-j2\pi f_L t}$$

Magnitude (M_0) *Phase* ($e^{-j2\pi f_L t}$)

A measured MRI signal is the sum of signals from everywhere in space



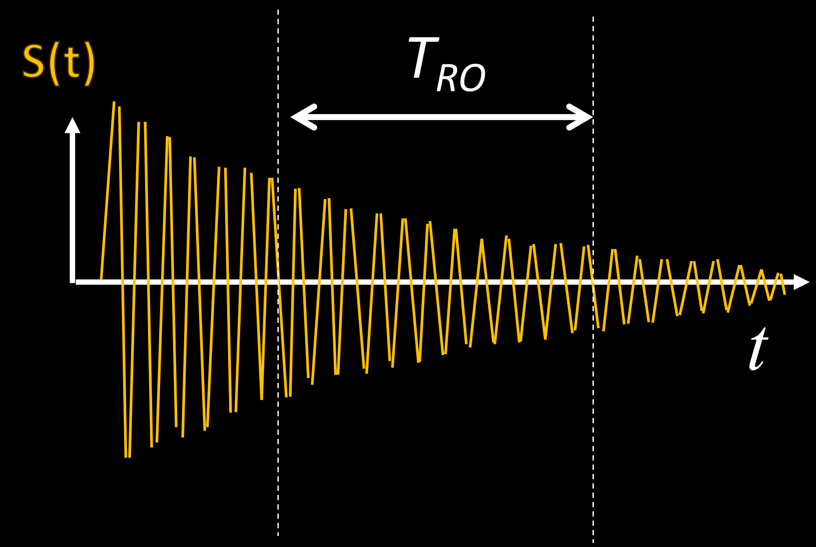
A measured MRI signal is the sum of signals from everywhere in space



$$S(t) = \iiint_{x,y,z} d^3x \cdot M_{xy}(x, y, z, t)$$

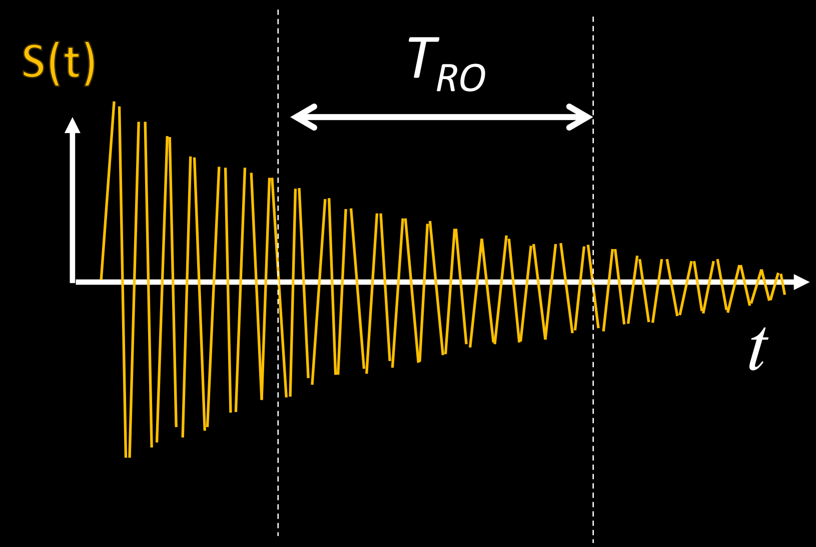
MRI signals take time to measure

Measure precession over a "Readout" period T_{RO}

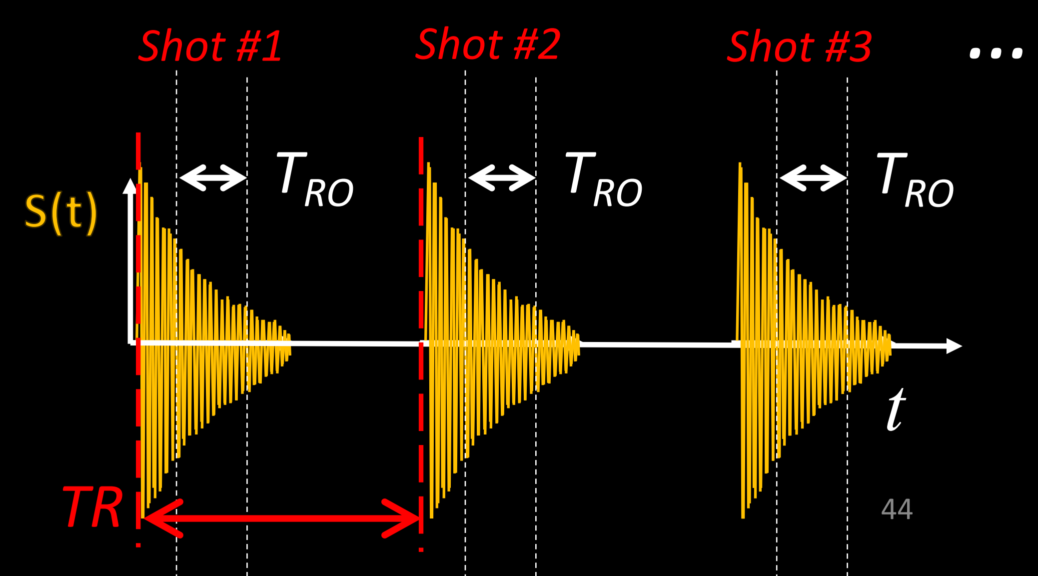


MRI signals take time to measure

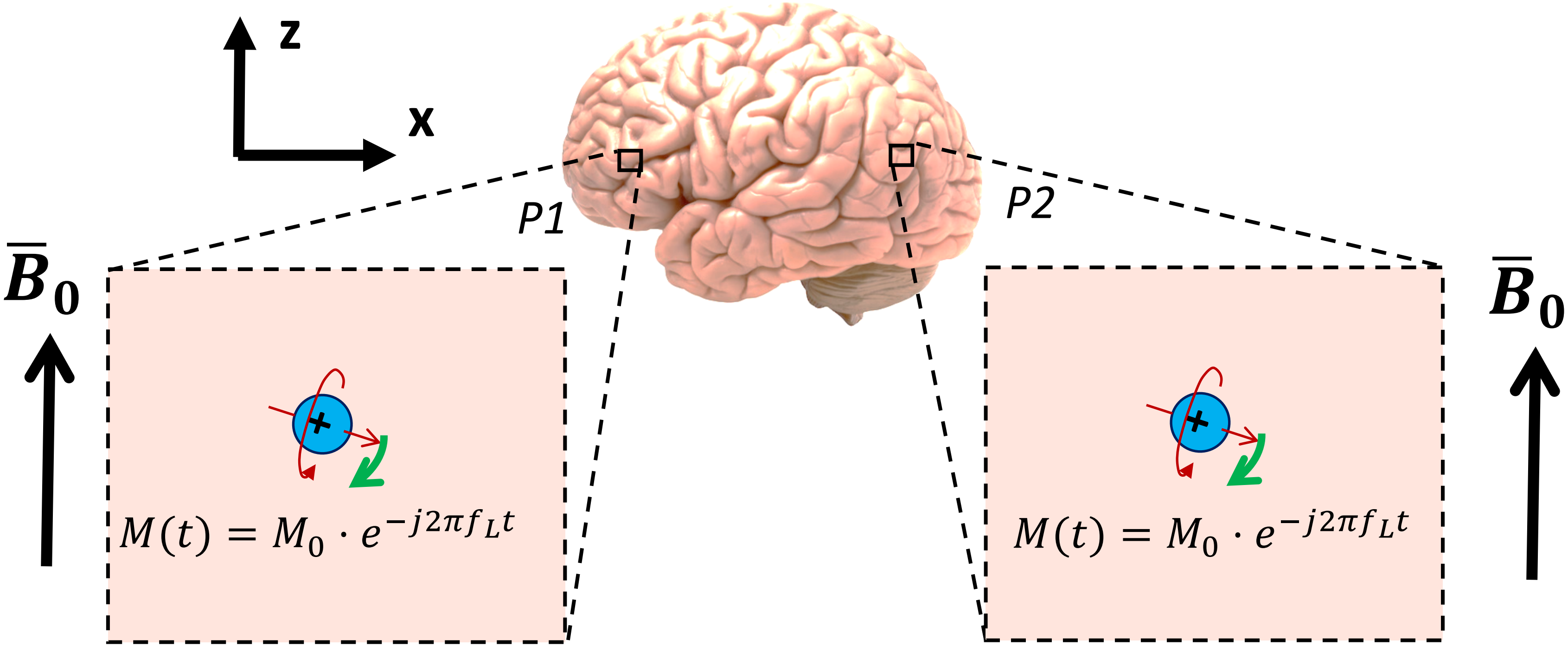
Measure precession over a "Readout" period T_{RO}



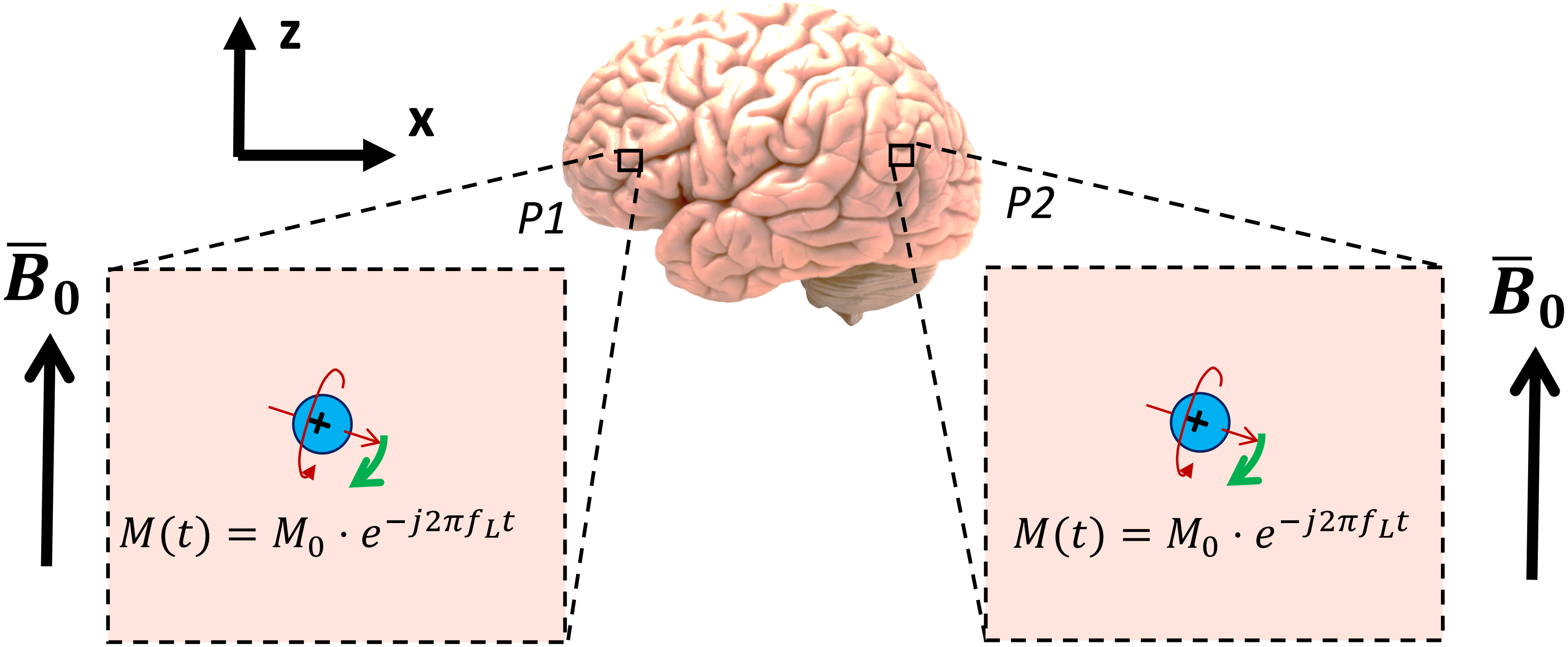
Acquiring sufficient data to form an image takes many iterations ("shots"), spaced by the "Repetition Time" (TR)



A Uniform B_0 Gives no Spatial Information*

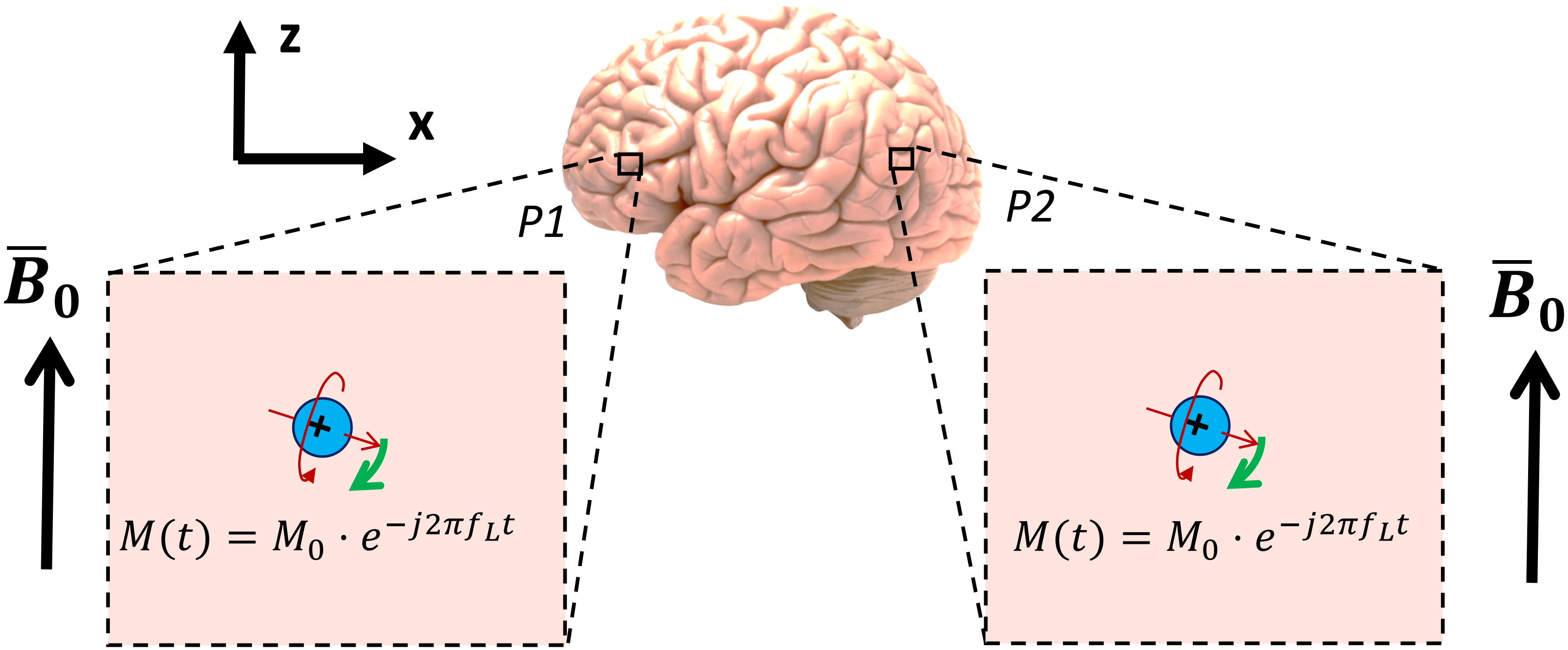


A Uniform B_0 Gives no Spatial Information*

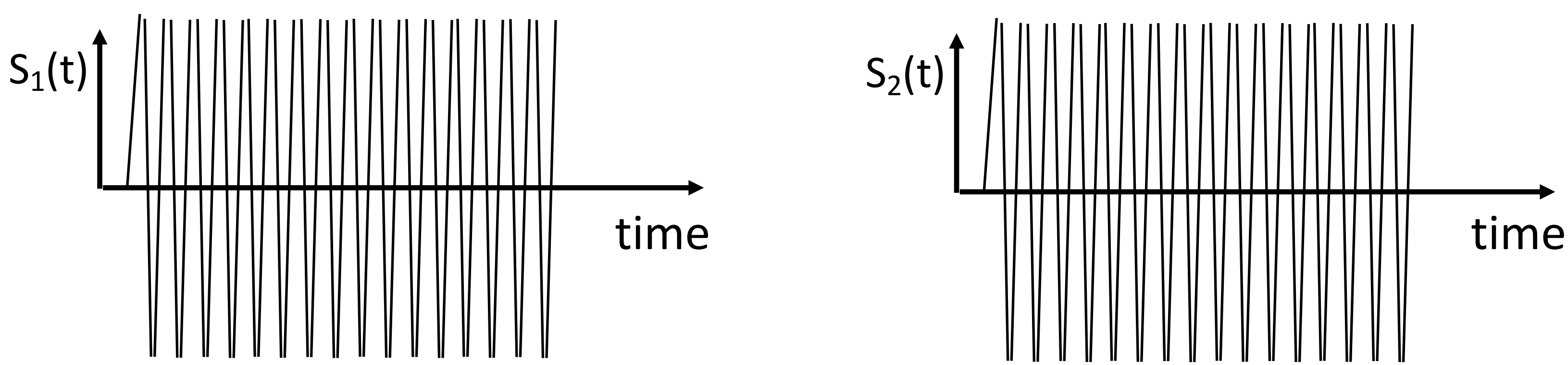


$$f_L = \gamma \cdot B$$

A Uniform B_0 Gives no Spatial Information*



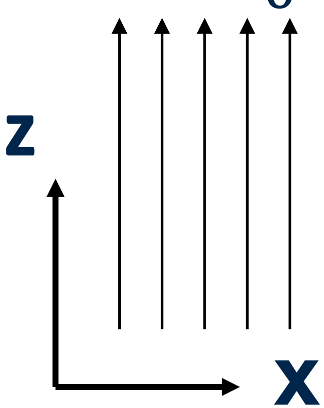
$$f_L = \gamma \cdot B$$



MR Imaging uses inhomogeneous "Gradient Fields" for spatial encoding

Uniform
magnetic field

B_0~3 T

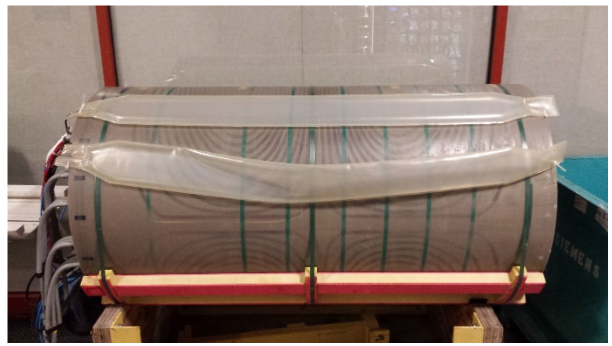


MR Imaging uses inhomogeneous "Gradient Fields" for spatial encoding

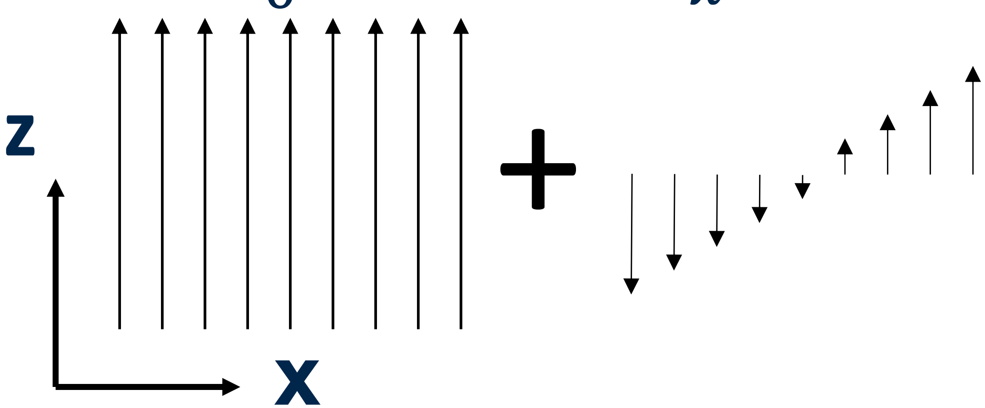
Uniform magnetic field

$B_0 \sim 3$ T

Field from "gradient" coils



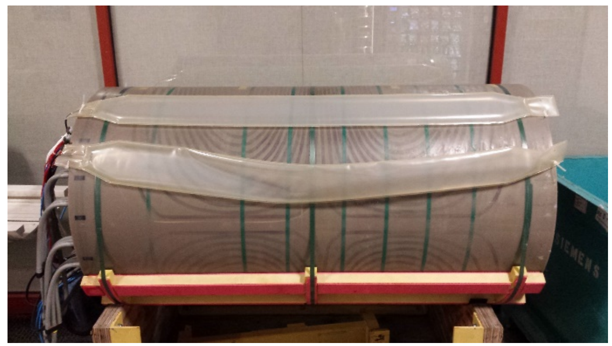
$G_x x \sim 10$ mT



MR Imaging uses inhomogeneous "Gradient Fields" for spatial encoding

Uniform magnetic field

Field from "gradient" coils

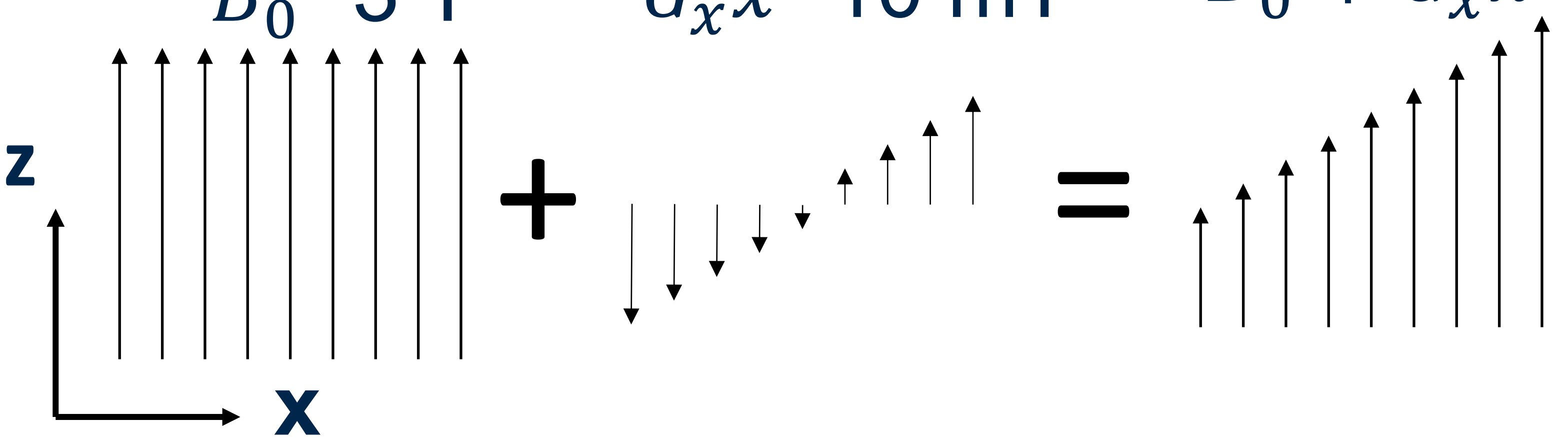


Total field

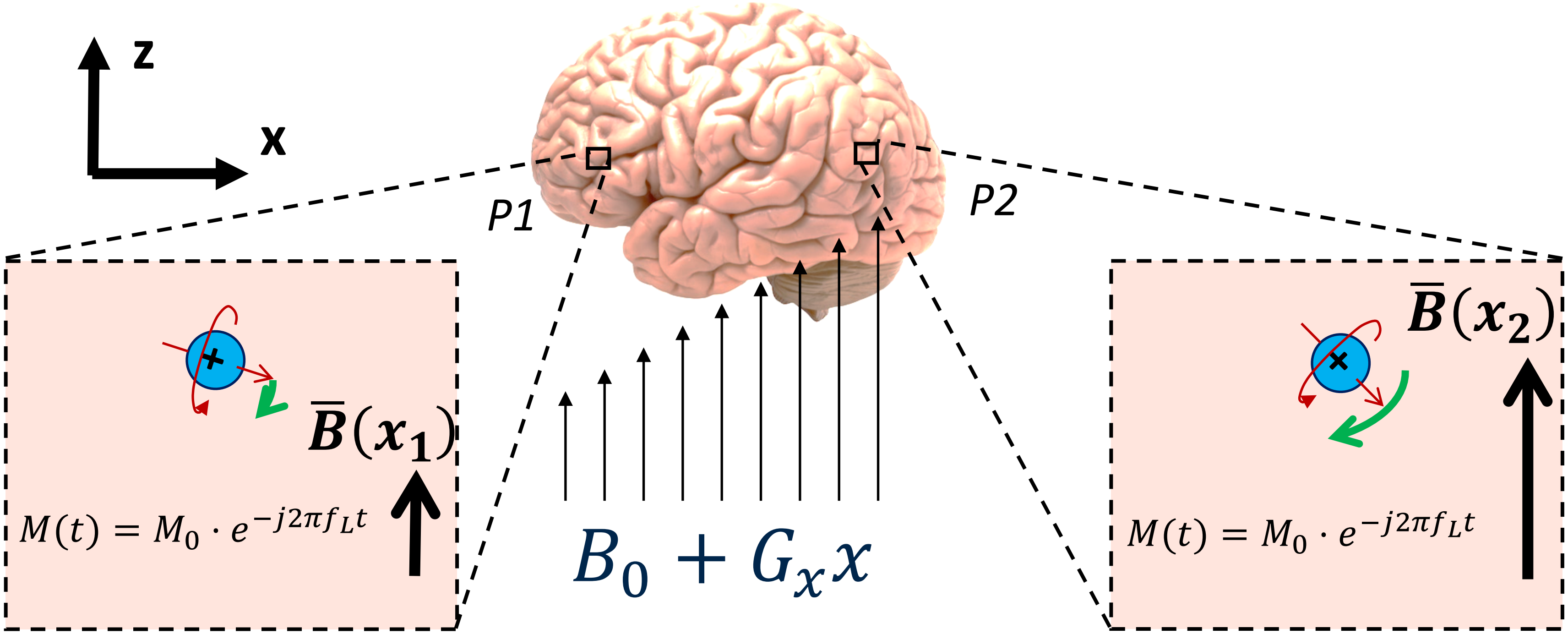
B_0~3 T

$G_x x$~10 mT

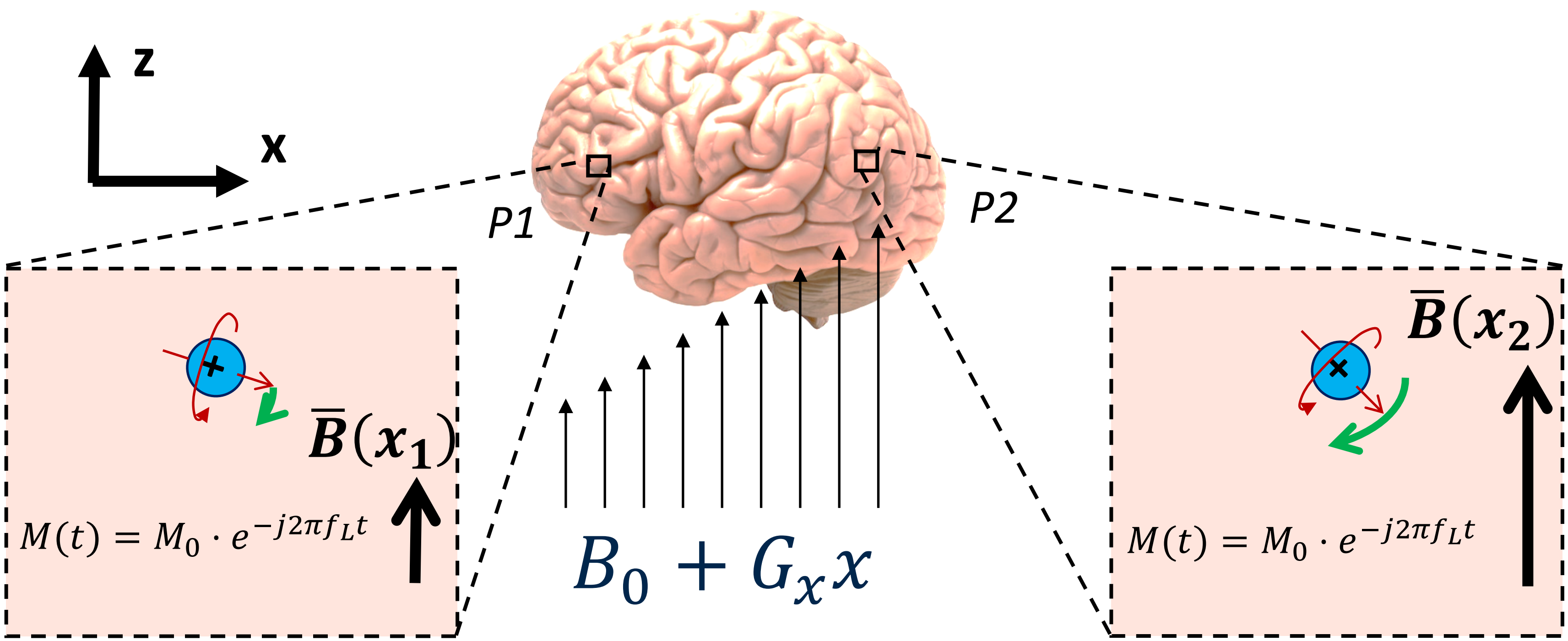
$B_0 + G_x x$



"Frequency Encoding" allows spatial encoding in one dimension

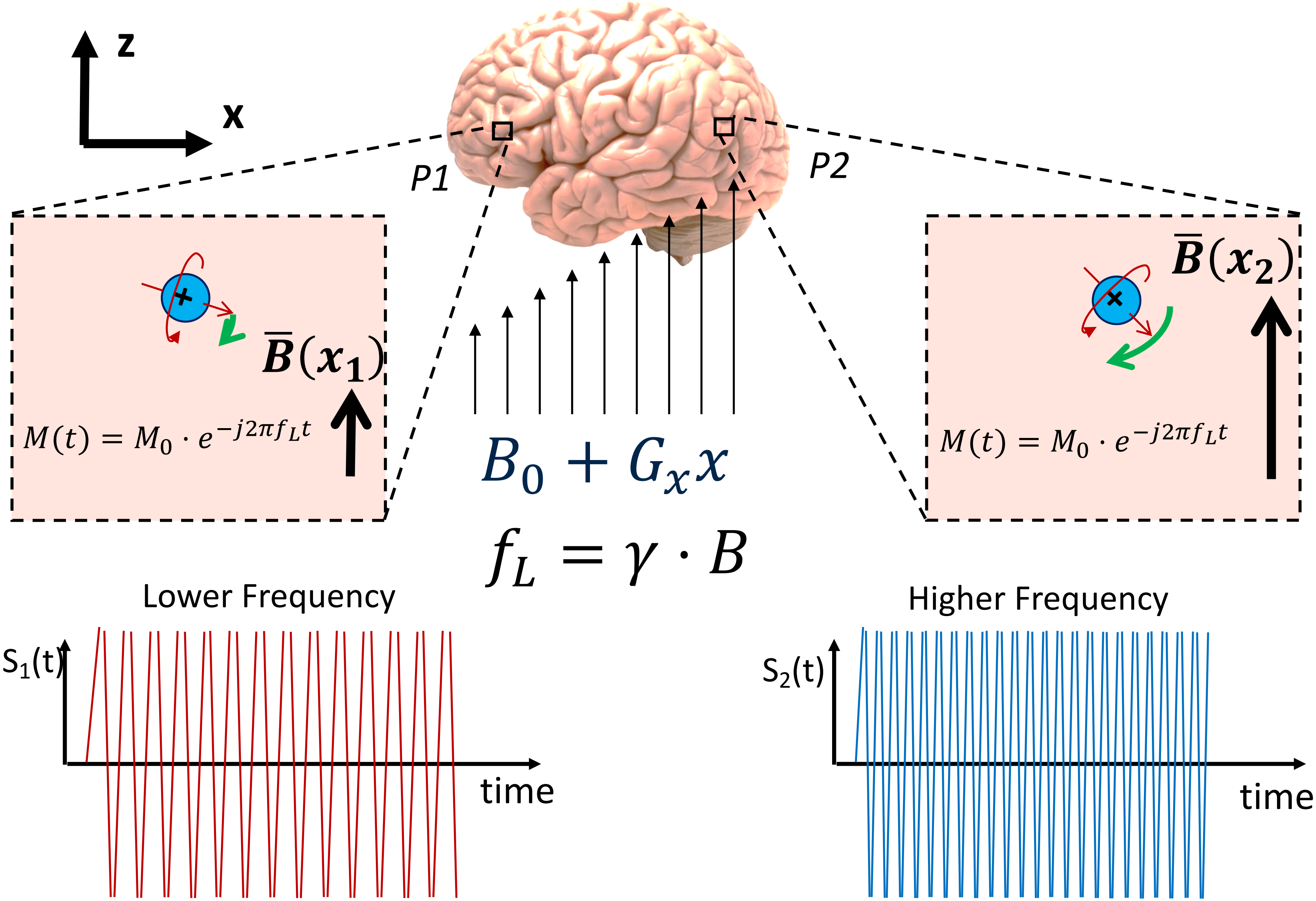


"Frequency Encoding" allows spatial encoding in one dimension

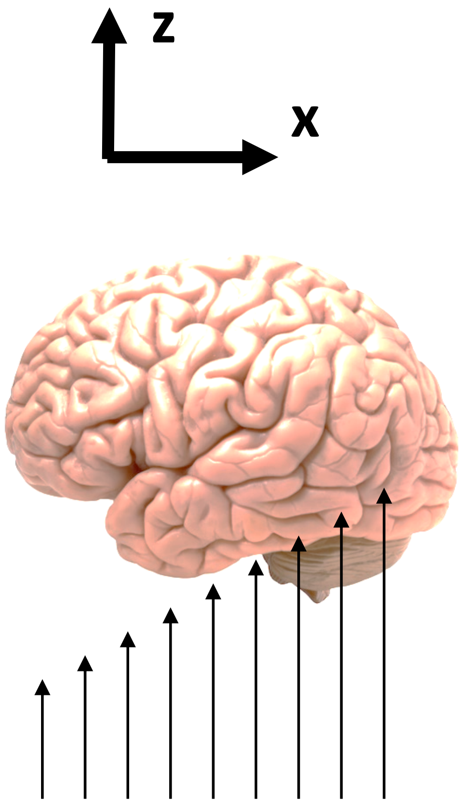


$$f_L = \gamma \cdot B$$

"Frequency Encoding" allows spatial encoding in one dimension



“Frequency Encoding” allows spatial encoding in one dimension

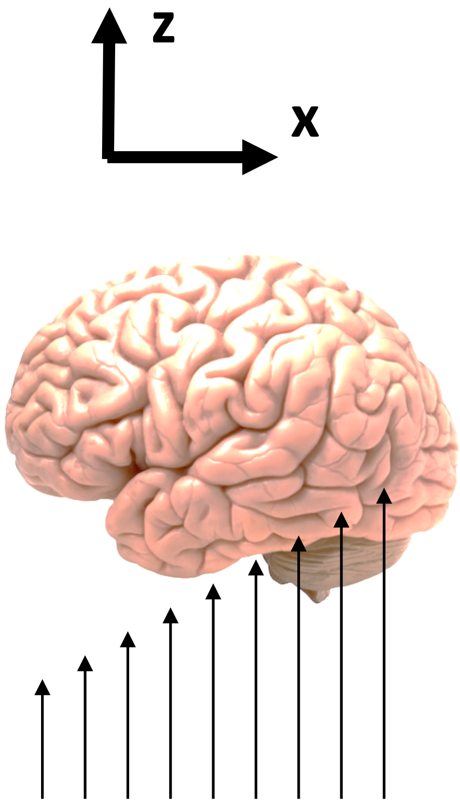


$$B_0 + G_x x$$

Measured Signal

$$S(t) = \iiint_{x,y,z} d^3x \cdot M_{xy}(x, y, z, t)$$

"Frequency Encoding" allows spatial encoding in one dimension



$B_0 + G_x x$

Measured Signal

$$S(t) = \iiint_{x,y,z} d^3x \cdot M_{xy}(x,y,z,t)$$

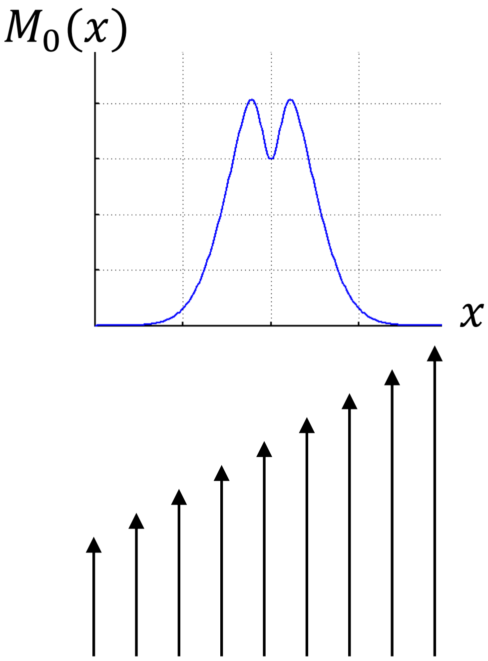
$$= \iiint_{x,y,z} d^3x \cdot M_0(x,y,z) \cdot e^{-j2\pi\gamma(B_0 + G_x \cdot x)t}$$

$$= e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x,y,z) \cdot e^{-j2\pi\gamma G_x x t}$$

Desired Image

Frequency Encoding: A 1D Example

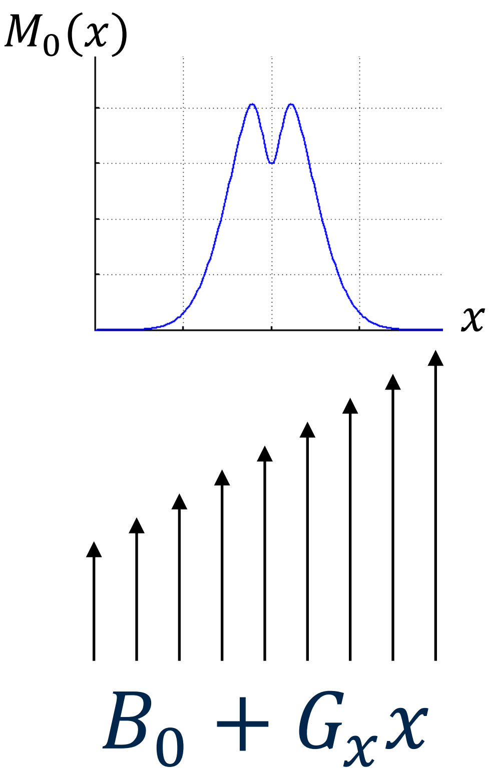
1D Object



$$B_0 + G_x x$$

Frequency Encoding: A 1D Example

1D Object



Measured Signal

$$S(t) = \int_x dx \cdot S(x,t)$$

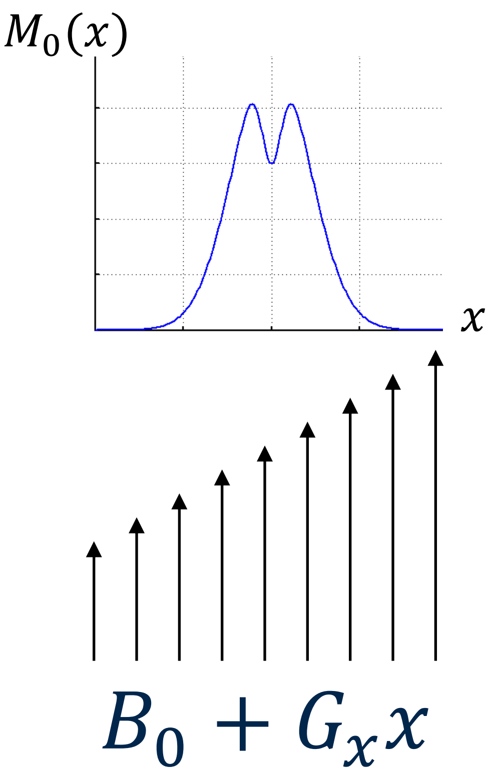
$$= \int_x dx \cdot M_0(x) \cdot e^{j2\pi\gamma(B_0 + G_x x)t}$$

$$= e^{-j2\pi\gamma B_0 t} \cdot \int_x dx \cdot M_0(x) \cdot e^{j2\pi\gamma G_x \cdot x \cdot t}$$

Desired Image

Frequency Encoding: A 1D Example

1D Object



Measured Signal

$$S(t) = \int_x dx \cdot S(x,t)$$

$$= \int_x dx \cdot M_0(x) \cdot e^{j2\pi\gamma(B_0 + G_x x)t}$$

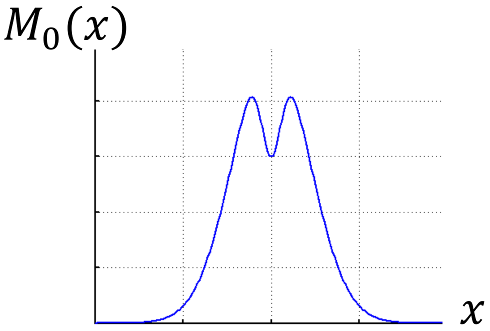
Fourier Transform of $M_0(x)$

$$= e^{-j2\pi\gamma B_0 t} \cdot \int_x dx \cdot M_0(x) \cdot e^{j2\pi\gamma G_x \cdot x \cdot t}$$

Desired Image

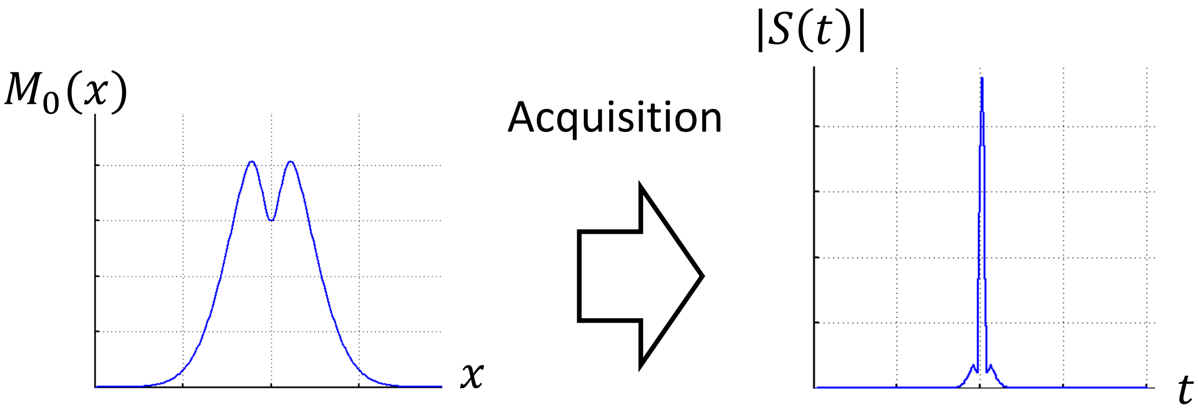
Reconstruct Image with Inverse Fourier Transform

1D Object



Reconstruct Image with Inverse Fourier Transform

1D Object *Measured Signal*

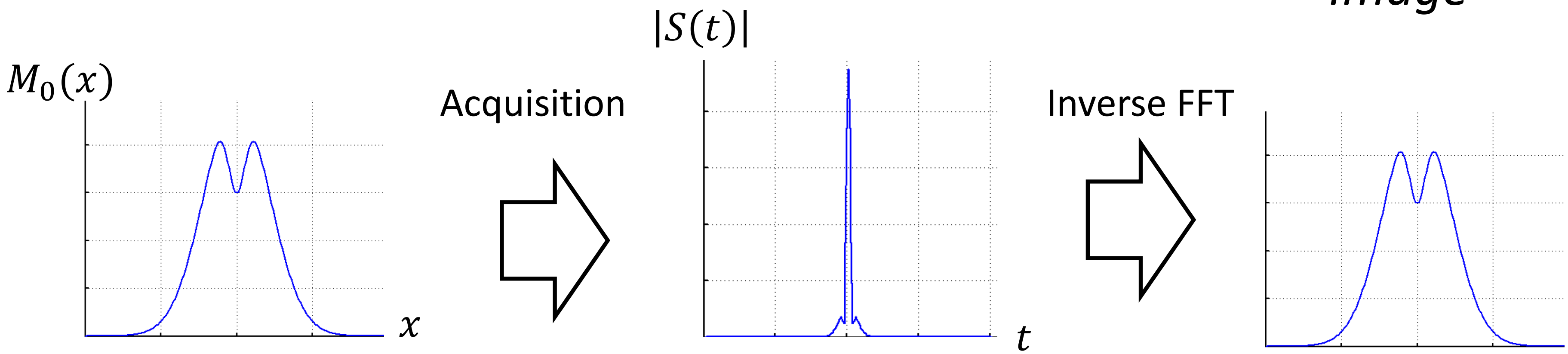


Fourier Transform of $M_0(x)$

$$S(t) = e^{-j2\pi\gamma B_0 t} \cdot \int_x dx \cdot M_0(x) \cdot e^{j2\pi\gamma G_x \cdot x \cdot t}$$

Reconstruct Image with Inverse Fourier Transform

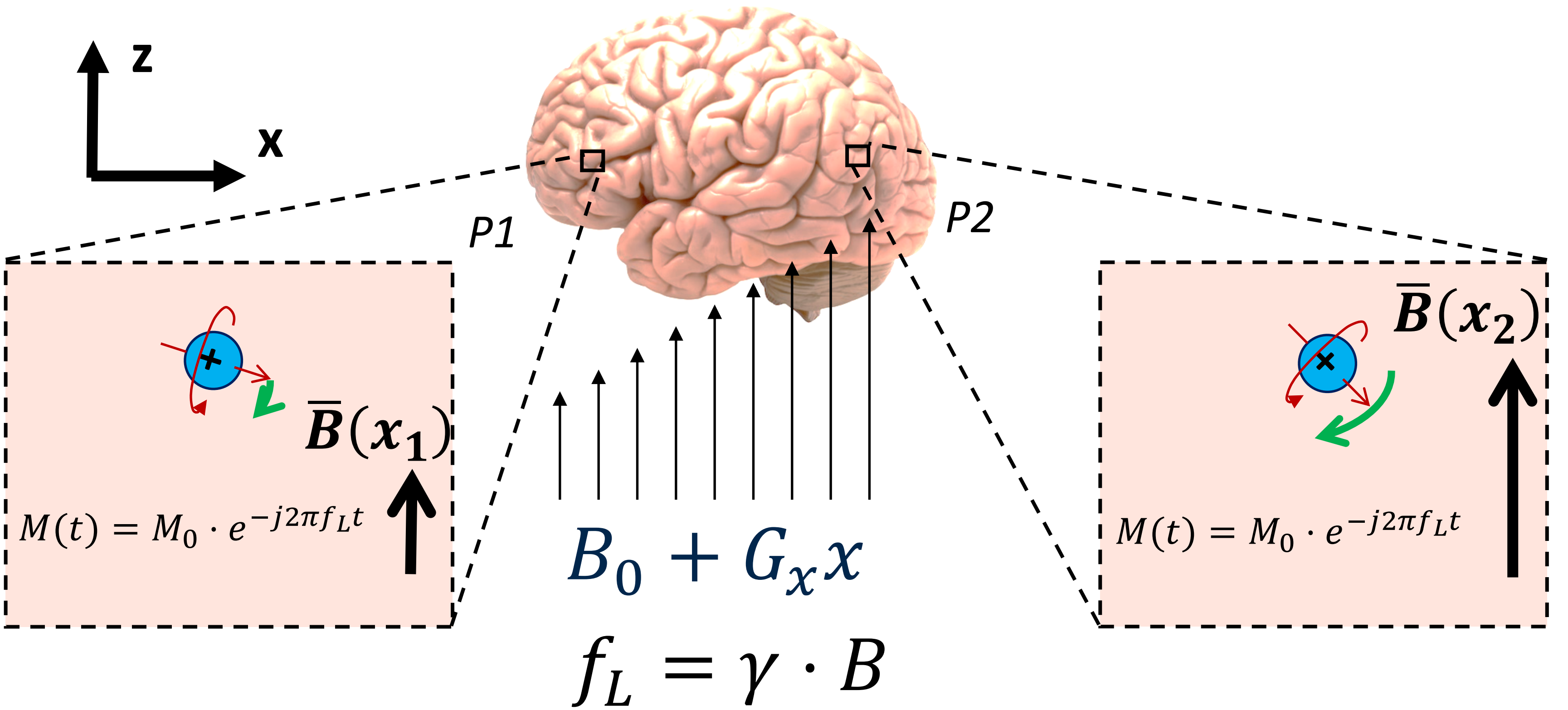
1D Object *Measured Signal* *Reconstructed Image*



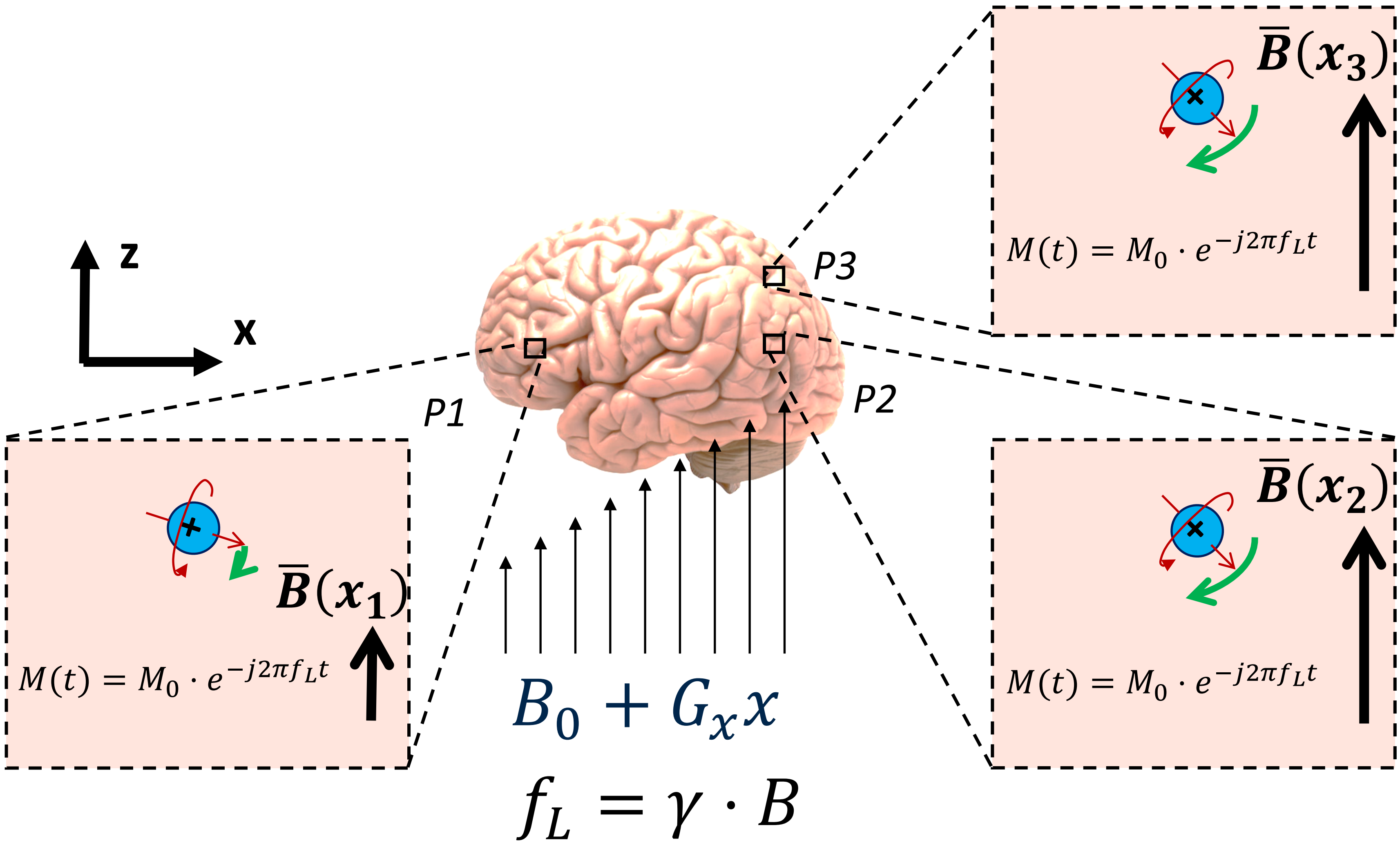
Fourier Transform of $M_0(x)$

$$S(t) = e^{-j2\pi\gamma B_0 t} \cdot \int_x dx \cdot M_0(x) \cdot e^{j2\pi\gamma G_x \cdot x \cdot t}$$

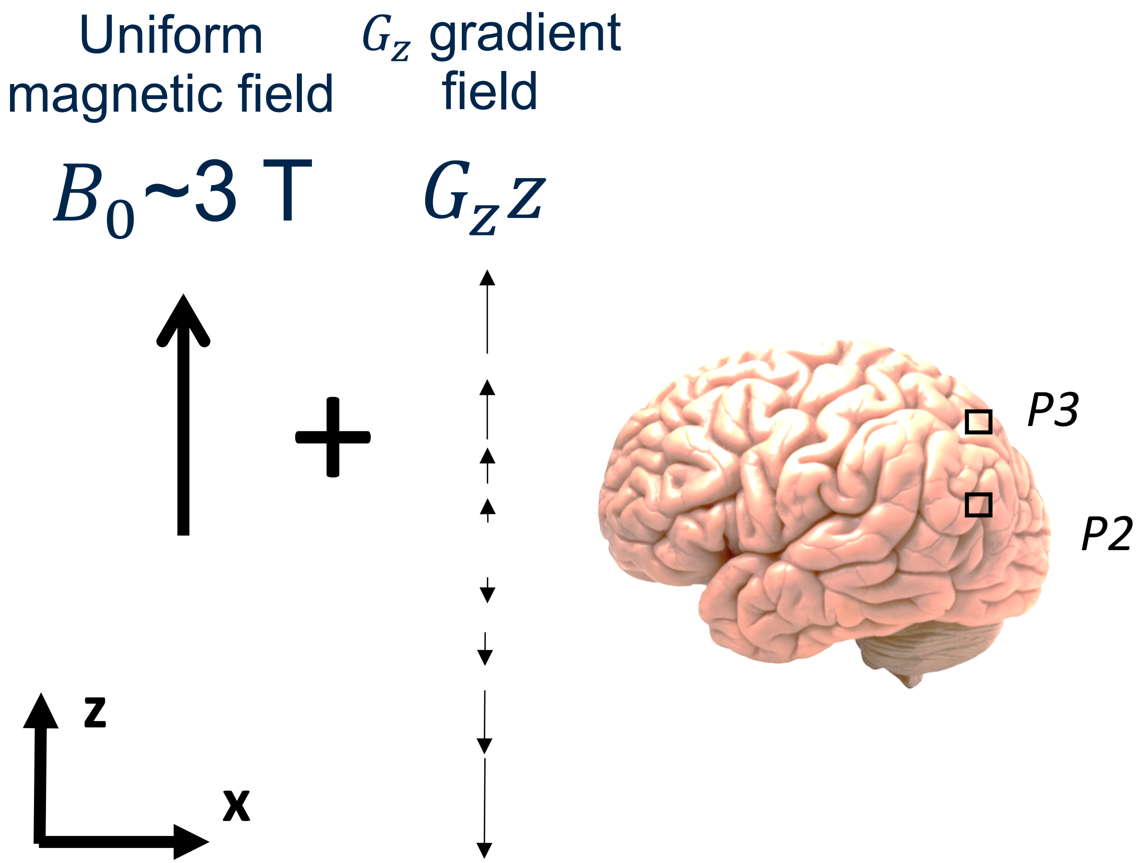
Frequency Encoding cannot encode along multiple dimensions*



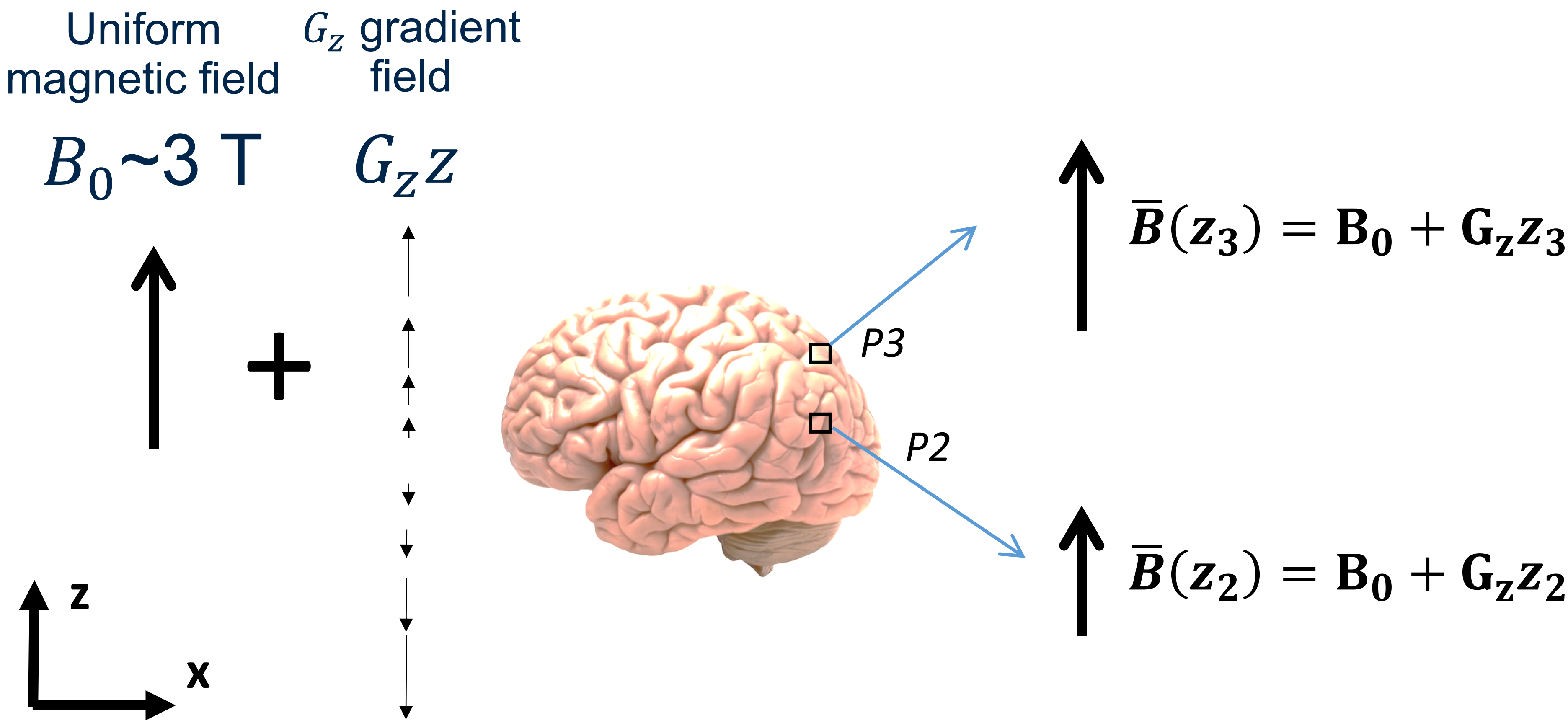
Frequency Encoding cannot encode along multiple dimensions*



A second gradient field allows encoding along other directions



A second gradient field allows encoding along other directions

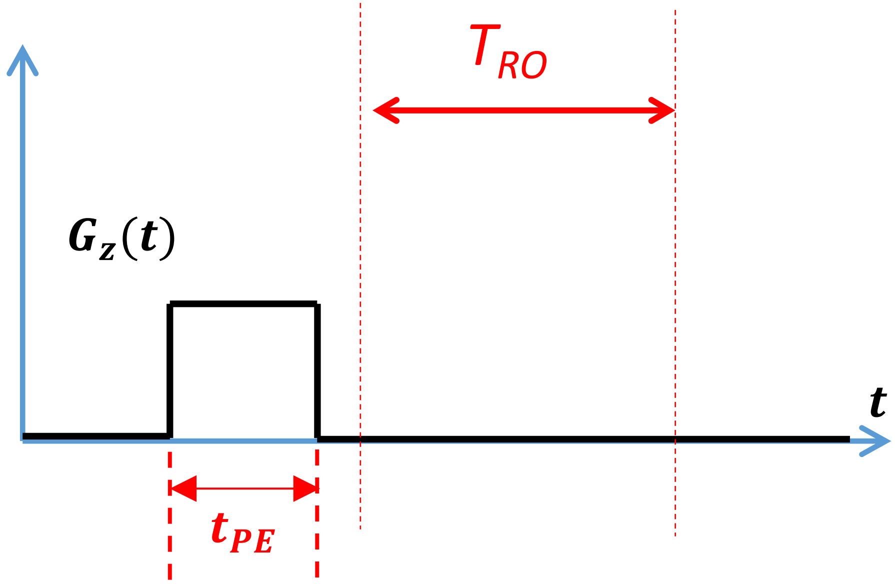


“Phase Encode” gradients are turned on for short “blips”

Phase Encode gradient *:*

Blipped

Used prior to acquisition



"Phase Encode" gradients are turned on for short "blips"

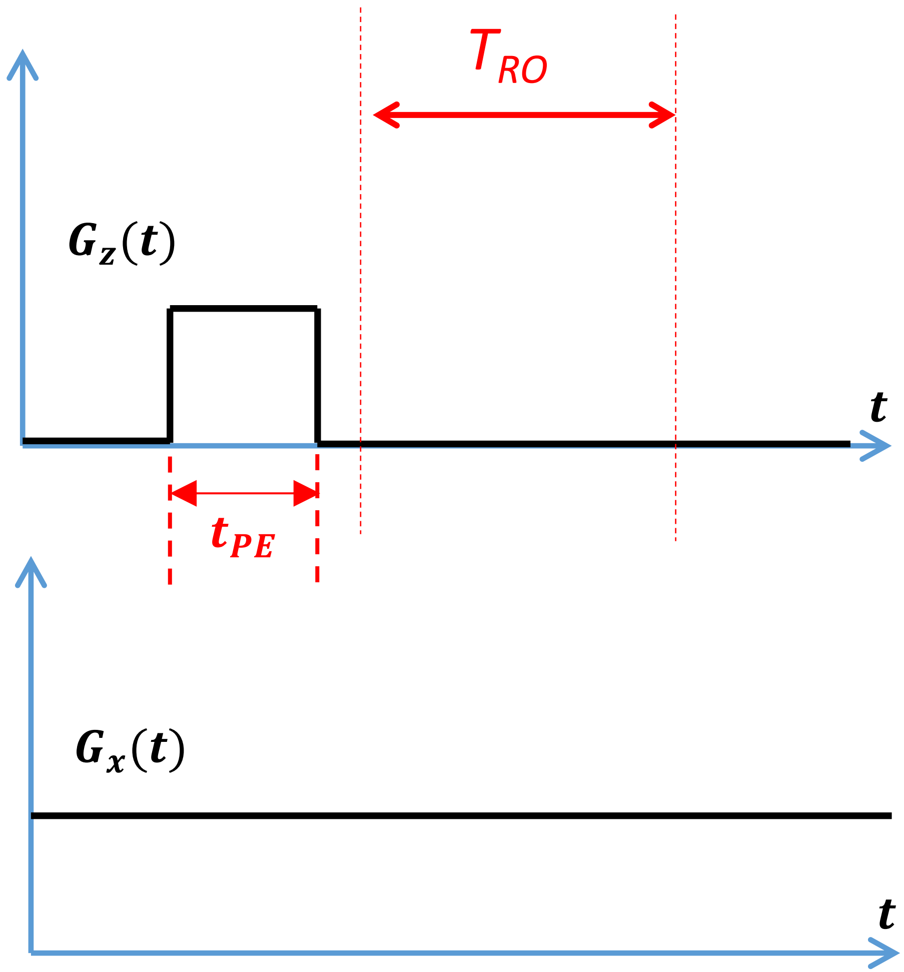
Phase Encode gradient *:*

Blipped

Used prior to acquisition

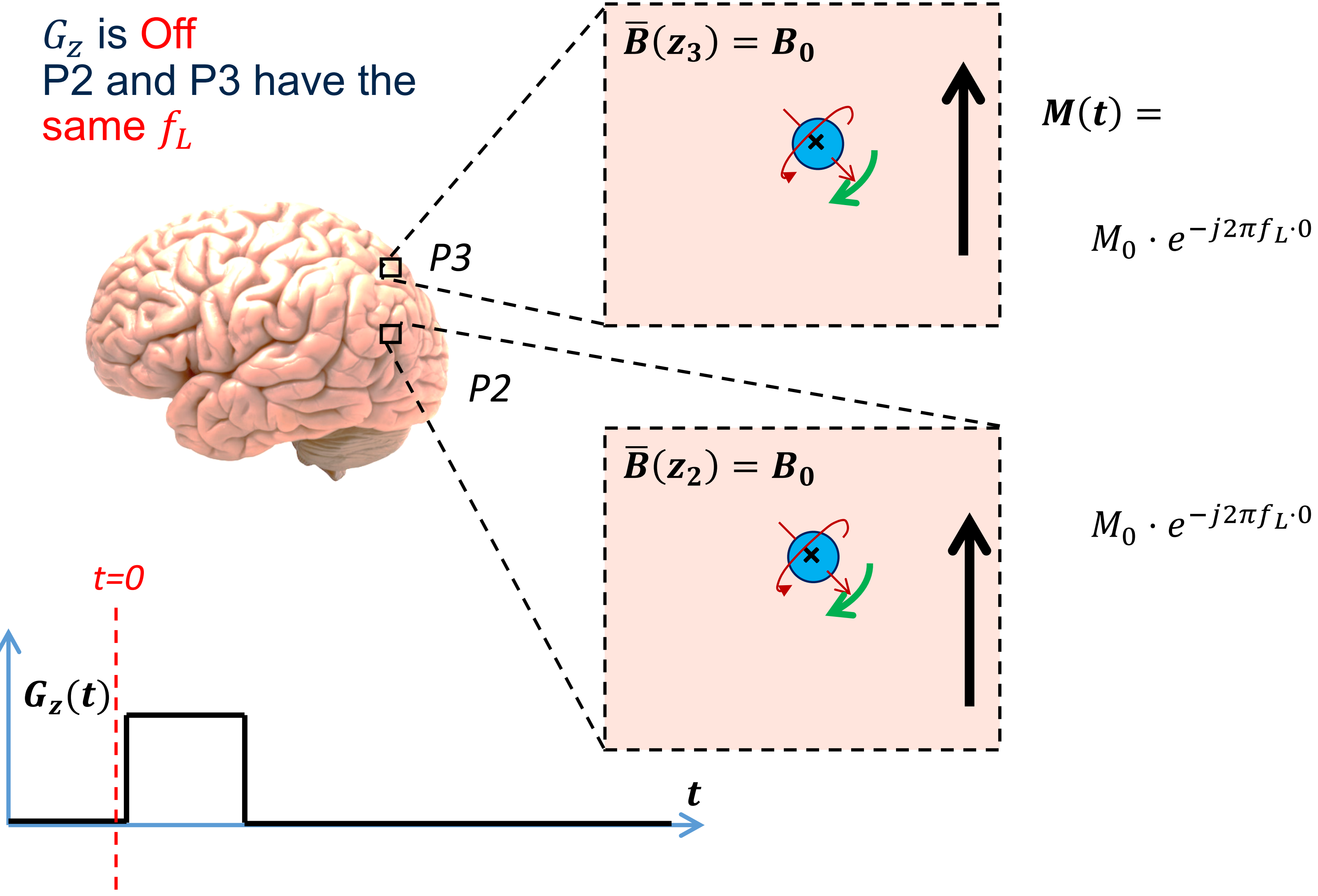
Frequency Encode gradient :

Always on during data acquisition



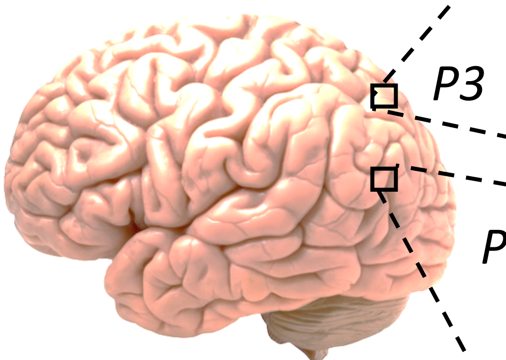
Different z-locations acquire different phases due to the blip

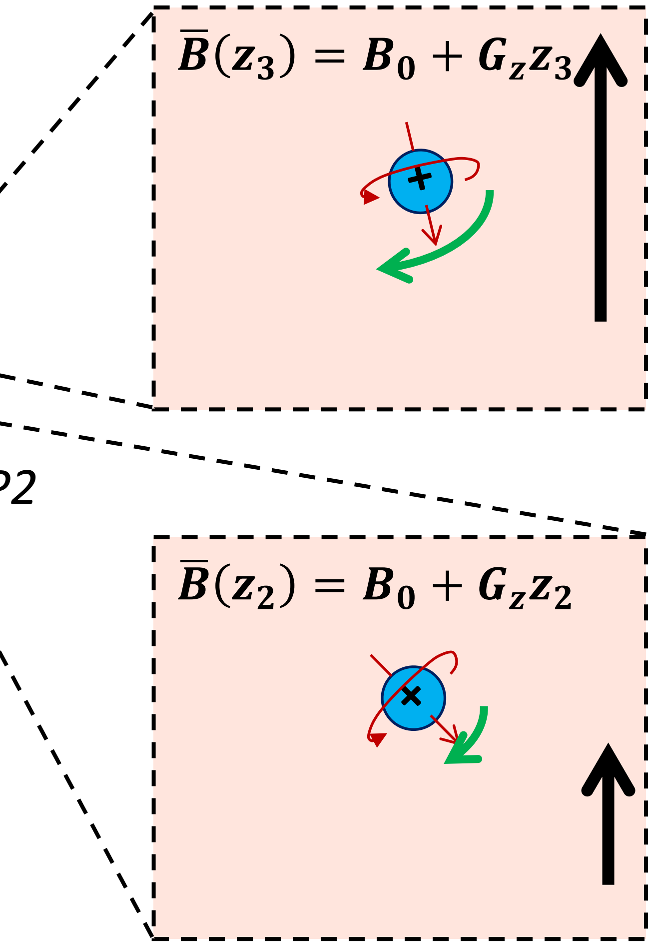
- G_z is Off
- P2 and P3 have the same f_L



Different z-locations acquire different phases due to the blip

- G_z is On
- P2 and P3 have different f_L

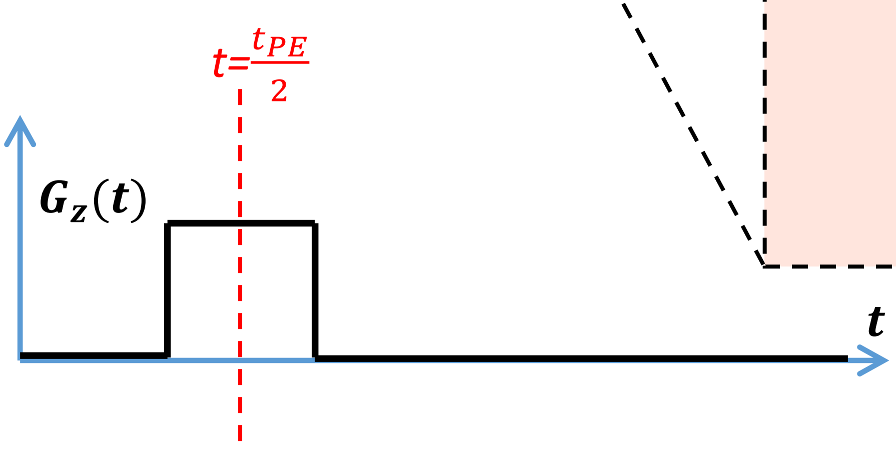




$M(t) =$

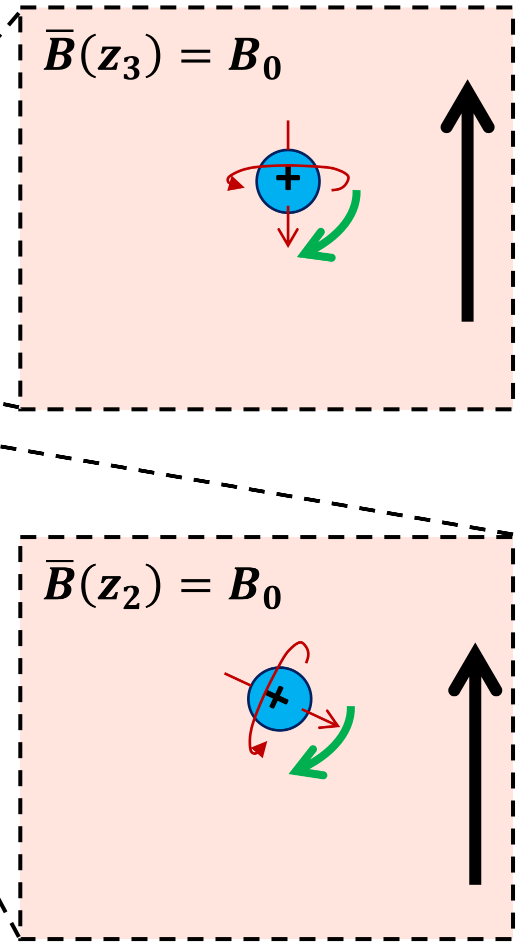
$M_0 \cdot e^{-j2\pi\gamma(B_0+G_z z_3)t_{PE}/2}$

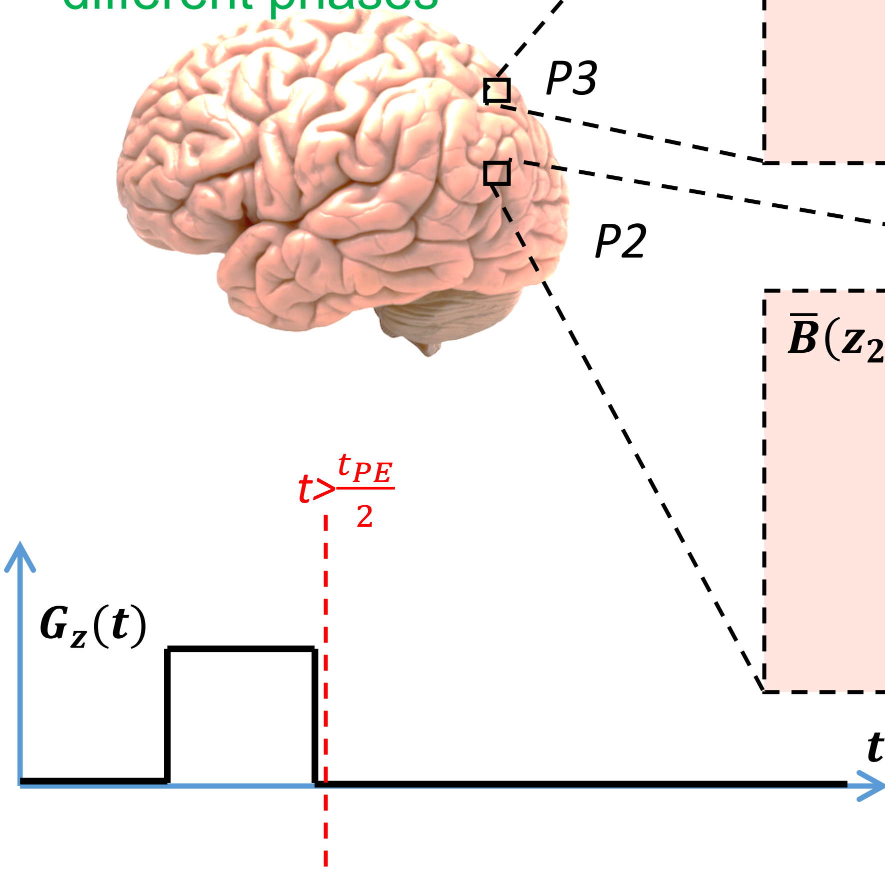
$M_0 \cdot e^{-j2\pi\gamma(B_0+G_z z_2)t_{PE}/2}$



Different z-locations acquire different phases due to the blip

- G_z is Off
- P2 and P3 have the same f_L
- P2 and P3 have different phases





$\boldsymbol{M(t)} =$

$M_0 \cdot e^{-j2\pi\gamma(G_z z_3)t_{PE}} \cdot e^{-j2\pi f_L t}$

Different Signal Phase

$$\boldsymbol{\varphi_{PE} = 2\pi\gamma z \cdot G_z t_{PE}}$$

$M_0 \cdot e^{-j2\pi\gamma(G_z z_2)t_{PE}} \cdot e^{-j2\pi f_L t}$

Different z-locations acquire different phases due to the blip

Measured Signal

$$\overbrace{S(t)} = \iiint_{x,y,z} d^3x \cdot M_{xy}(x, y, z, t)$$

Different z-locations acquire different phases due to the blip

Measured Signal

$$\overbrace{S(t)} = \iiint_{x,y,z} d^3x \cdot M_{xy}(x, y, z, t)$$

$$= \iiint_{x,y,z} d^3x \cdot M_0(x, y, z) \cdot e^{-j2\pi\gamma(G_z z)t_{PE}} \cdot e^{-j2\pi\gamma B_0 t}$$

$$= e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x, y, z) \cdot e^{-j2\pi\gamma z \cdot G_z t_{PE}}$$

Different z-locations acquire different phases due to the blip

Measured Signal

$$\overbrace{S(t)} = \iiint_{x,y,z} d^3x \cdot M_{xy}(x,y,z,t)$$

$$= \iiint_{x,y,z} d^3x \cdot M_0(x,y,z) \cdot e^{-j2\pi\gamma(G_z z)t_{PE}} \cdot e^{-j2\pi\gamma B_0 t}$$

NOT the Fourier Transform of $M_0(z)$

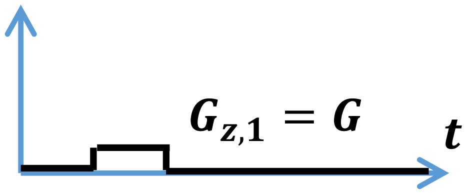
$$= e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x,y,z) \cdot e^{-j2\pi\gamma z \cdot G_z t_{PE}}$$

Vary Phase Encode blip size across multiple shots to acquire sufficient data

Phase Encode blip

Acquired Signal

Shot #1 :



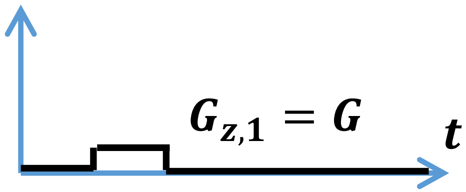
$$S_{PE1}(t) = e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x,y,z) \cdot e^{-j2\pi\gamma z \cdot G t_{PE}}$$

Vary Phase Encode blip size across multiple shots to acquire sufficient data

Phase Encode blip

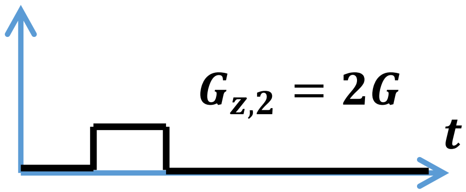
Acquired Signal

Shot #1 :



$$S_{PE1}(t) = e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x,y,z) \cdot e^{-j2\pi\gamma z \cdot G t_{PE}}$$

Shot #2 :

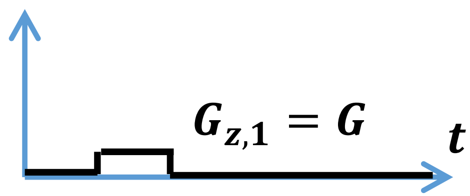


$$S_{PE2}(t) = e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x,y,z) \cdot e^{-j2\pi\gamma z \cdot 2G t_{PE}}$$

Vary Phase Encode blip size across multiple shots to acquire sufficient data

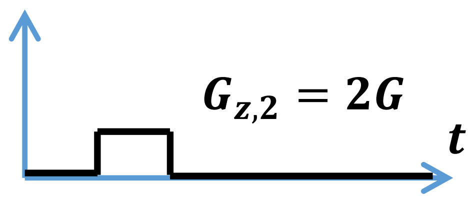
Phase Encode blip	Acquired Signal

Shot #1 :



$$S_{PE1}(t) = e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x,y,z) \cdot e^{-j2\pi\gamma z \cdot G t_{PE}}$$

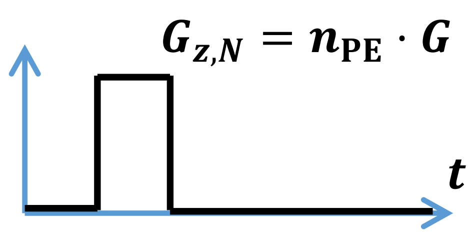
Shot #2 :



$$S_{PE2}(t) = e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x,y,z) \cdot e^{-j2\pi\gamma z \cdot 2G t_{PE}}$$

⋮

Shot #n_{PE} :



$$S_{PEN}(t) = e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x,y,z) \cdot e^{-j2\pi\gamma z \cdot n_{PE} G t_{PE}}$$

Treating n_{PE} as a variable turns the signal into a Fourier Transform

$$S(n_{PE}, t) = e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x, y, z) \cdot e^{-j2\pi\gamma z \cdot n_{PE} G t_{PE}}$$

Treating n_{PE} as a variable turns the signal into a Fourier Transform

$$S(n_{PE}, t) = e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x,y,z) \cdot e^{-j2\pi\gamma z \cdot n_{PE} G t_{PE}}$$

This IS the Fourier Transform of $M_0(z)$

Combining Phase Encoding with Frequency Encoding allows for 2D imaging

1D Phase Encoding

- ***Sample different n_{PE} across different shots***

$$S(n_{PE}, t) = e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x, y, z) \cdot e^{-j2\pi\gamma z \cdot n_{PE} G t_{PE}}$$

Combining Phase Encoding with Frequency Encoding allows for 2D imaging

1D Phase Encoding

- ***Sample different n_{PE} across different shots***

$$S(n_{PE}, t) = e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x, y, z) \cdot e^{-j2\pi\gamma z \cdot n_{PE} G t_{PE}}$$

1D Frequency Encoding

- ***Sample t at time points within one shot***

$$S(t) = e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x, y, z) \cdot e^{j2\pi\gamma G_x \cdot x \cdot t}$$

Combining Phase Encoding with Frequency Encoding allows for 2D imaging

1D Phase Encoding

- ***Sample different n_{PE} across different shots***

$$S(n_{PE}, t) = e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x, y, z) \cdot e^{-j2\pi\gamma z \cdot n_{PE} G t_{PE}}$$

1D Frequency Encoding

- ***Sample t at time points within one shot***

$$S(t) = e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x, y, z) \cdot e^{j2\pi\gamma G_x \cdot x \cdot t}$$

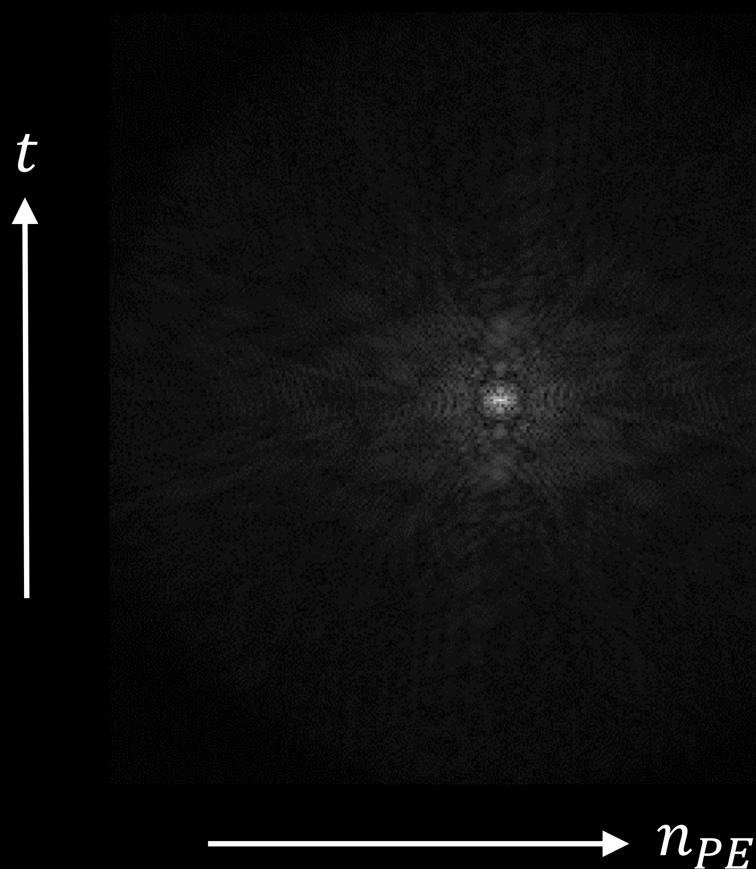
Combine: 2D Encoding

- ***Sample different n_{PE} across different shots***
- ***Sample t at time points within each shot***

$$S(n_{PE}, t) = e^{-j2\pi\gamma B_0 t} \cdot \iiint_{x,y,z} d^3x \cdot M_0(x, y, z) \cdot e^{-j2\pi\gamma z \cdot n_{PE} G t_{PE}} \cdot e^{j2\pi\gamma G_x \cdot x \cdot t}$$

The 2D Fourier Transform reconstructs an image from 2D sampled data

$$|S(n_{PE}, t)|^{\frac{1}{4}}$$

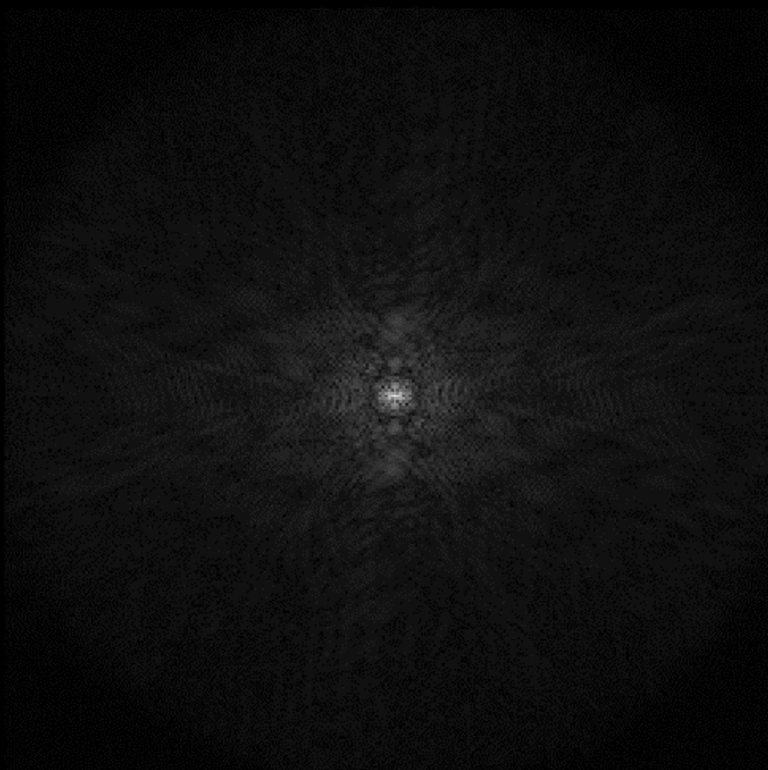


The 2D Fourier Transform reconstructs an image from 2D sampled data

$|S(n_{PE}, t)|^{\frac{1}{4}}$

Reconstructed Image

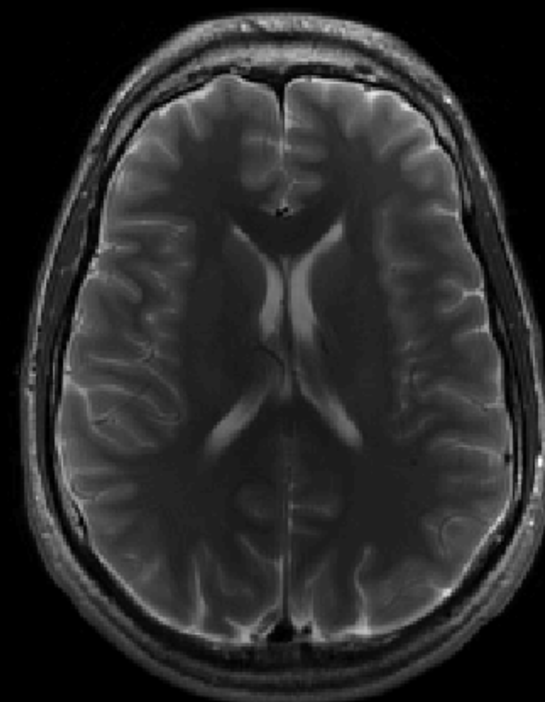
t



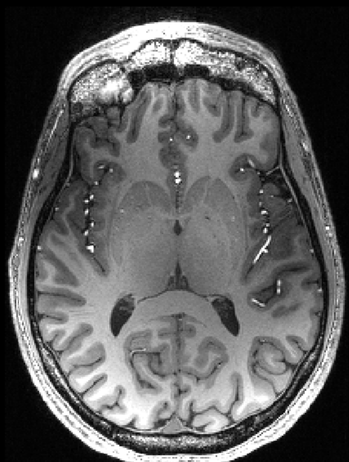
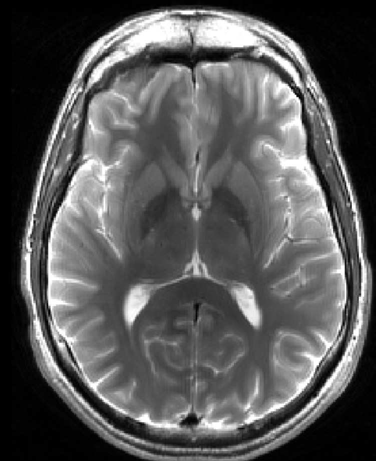
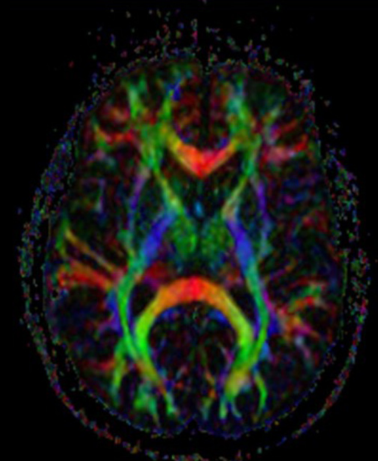
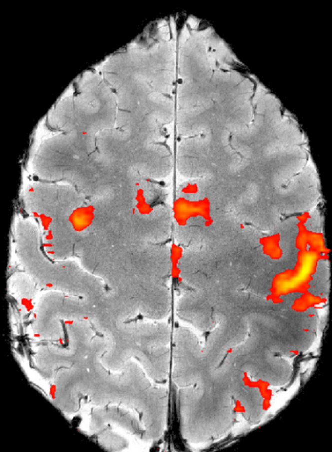
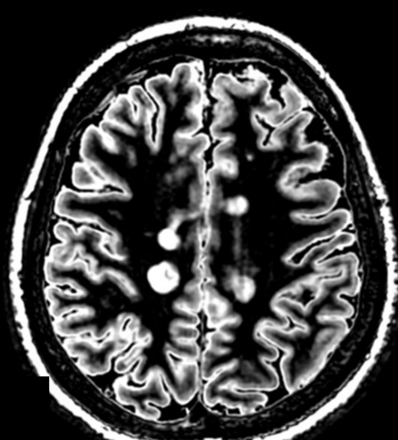
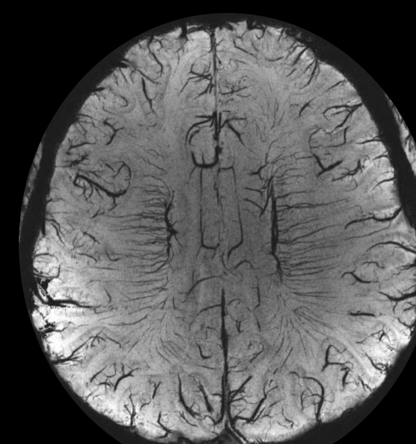
n_{PE}

Inverse
2D FFT

x

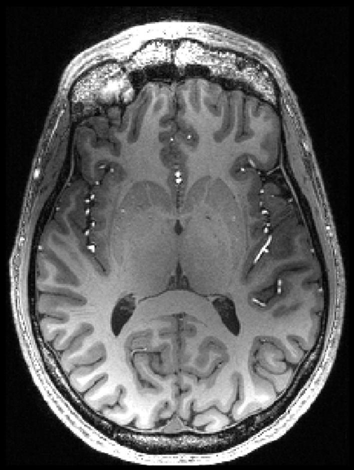


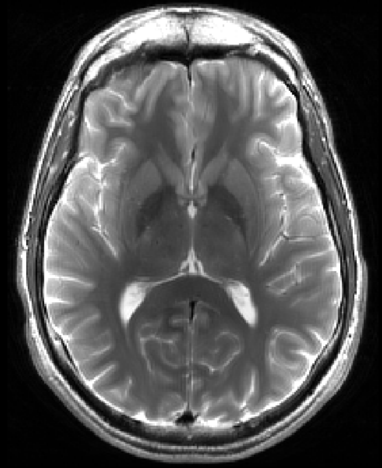
y

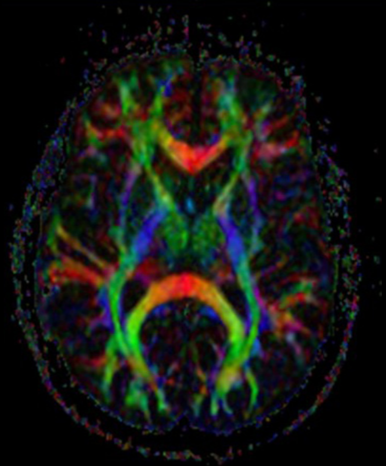







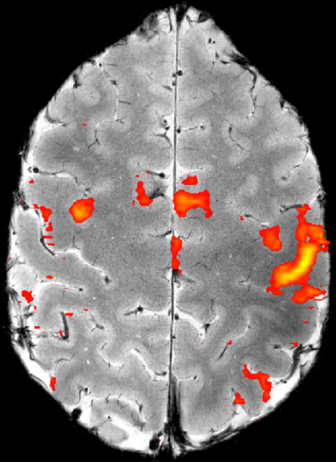
MPRAGE

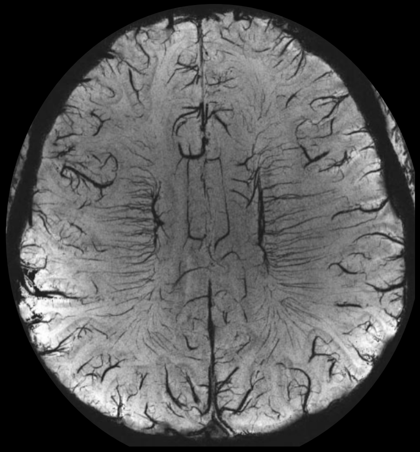
- T1
- Inflow effects



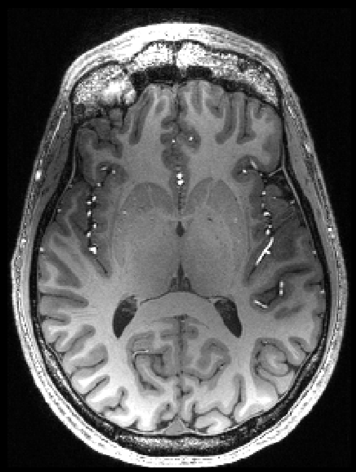




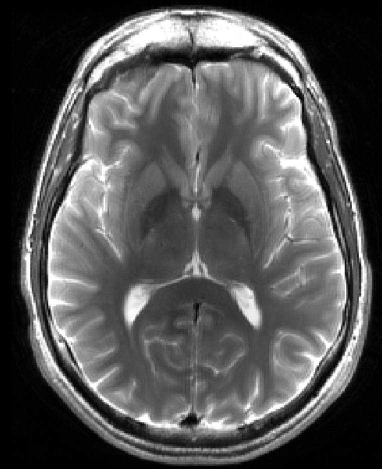


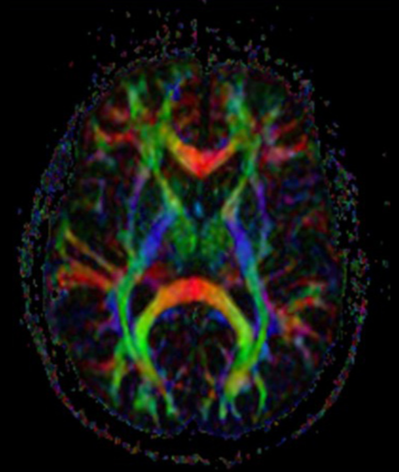


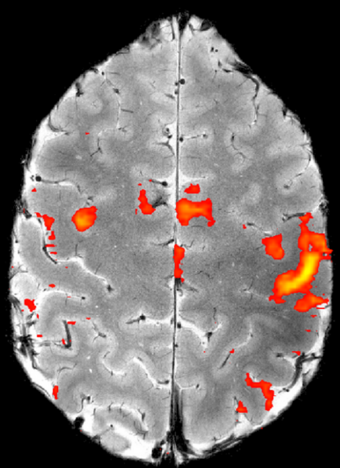
MPRAGE
- T1
- Inflow effects

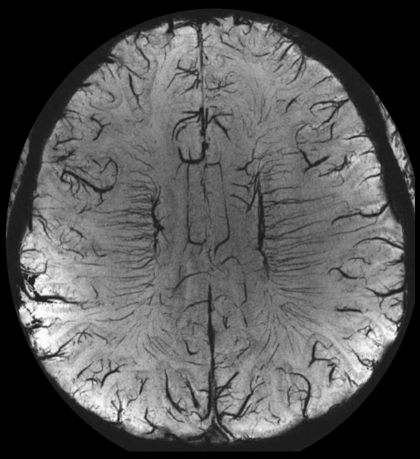


TSE
- T2
- Magnetization Transfer

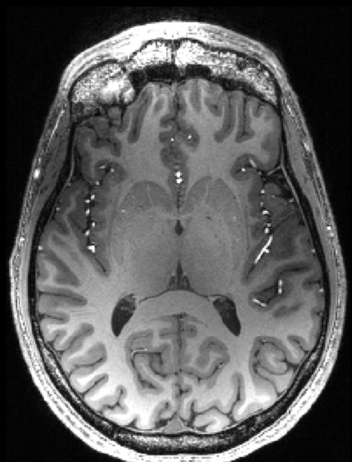




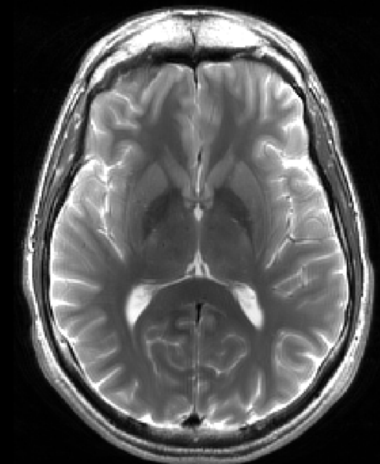




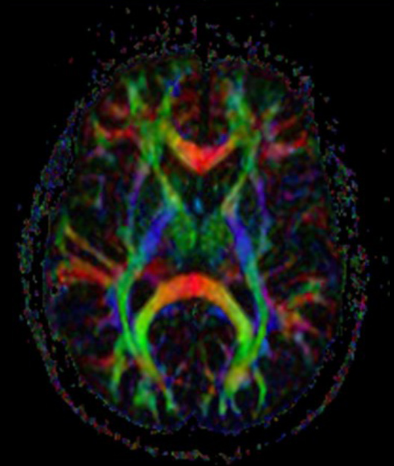
MPRAGE
- T1
- Inflow effects

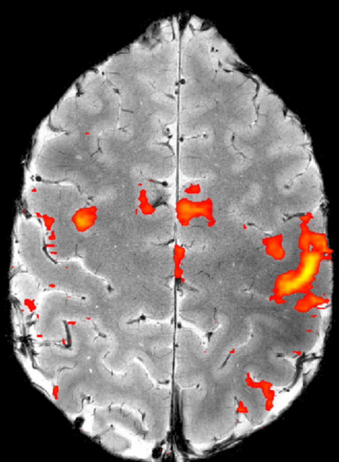


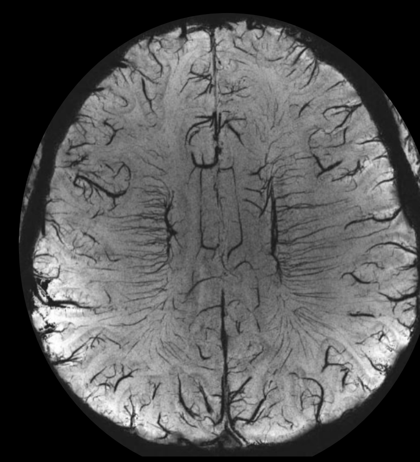
TSE
- T2
- Magnetization Transfer



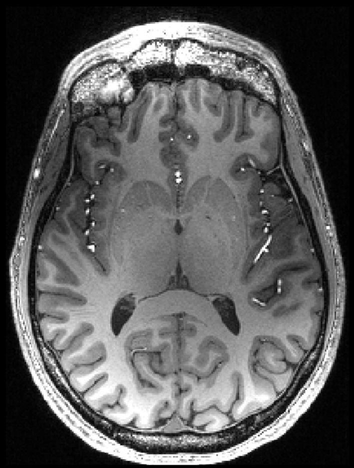
DTI
- Diffusion



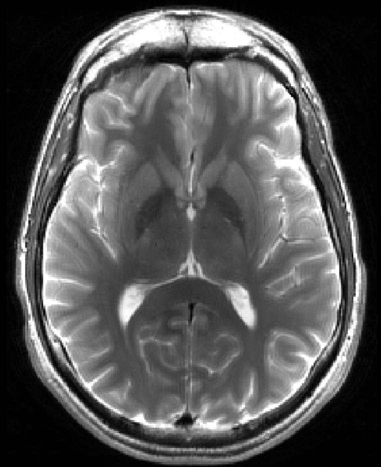




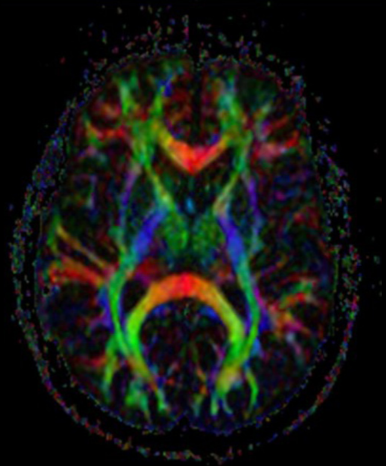
MPRAGE
- T1
- Inflow effects



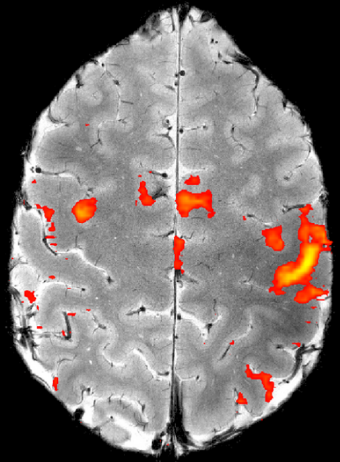
TSE
- T2
- Magnetization Transfer

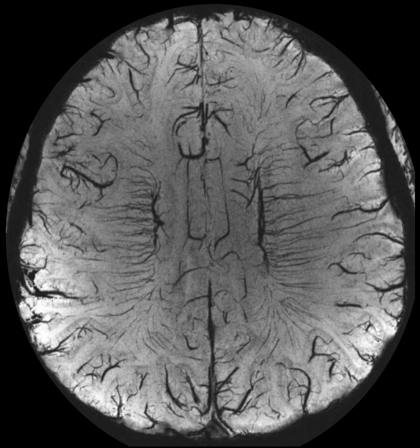


DTI
- Diffusion

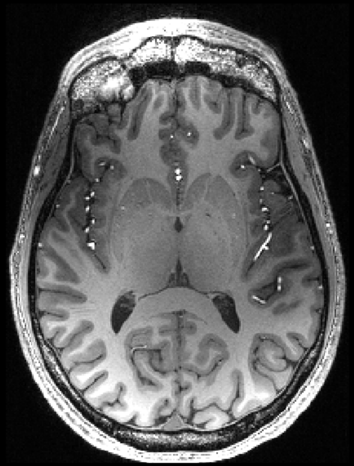


BOLD fMRI
- T2* or T2

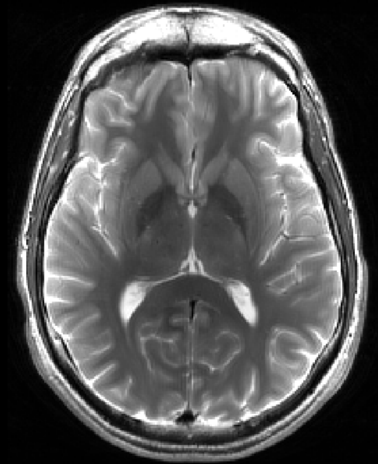




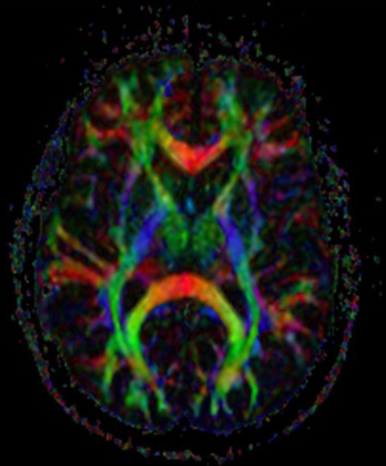
MPRAGE
- T1
- Inflow effects



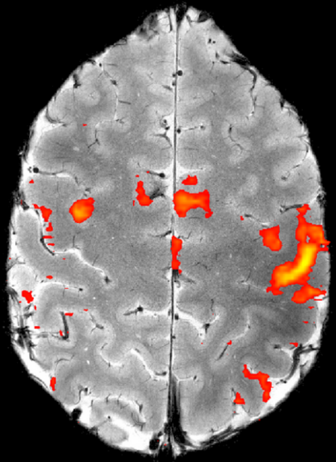
TSE
- T2
- Magnetization Transfer



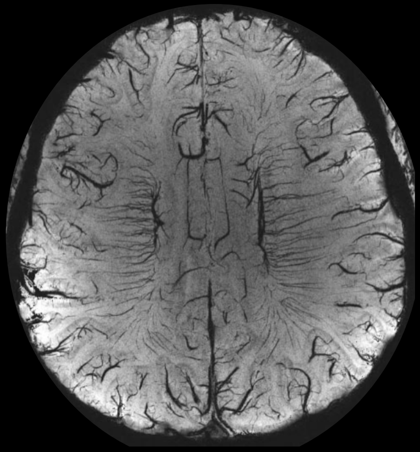
DTI
- Diffusion



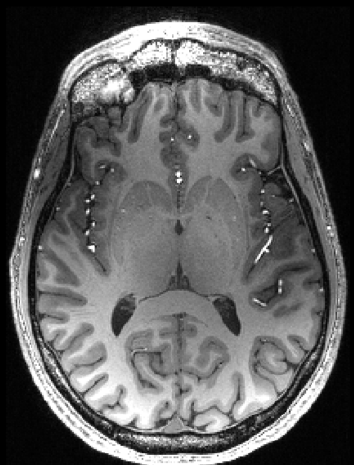
BOLD fMRI
- T2* or T2



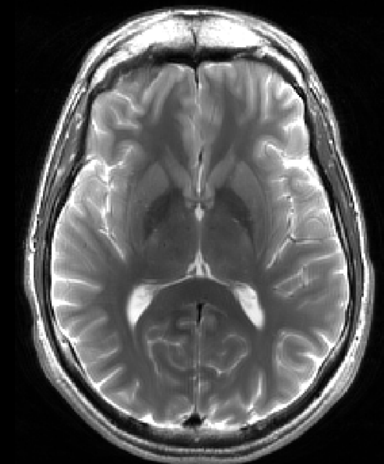
DIR
- T1



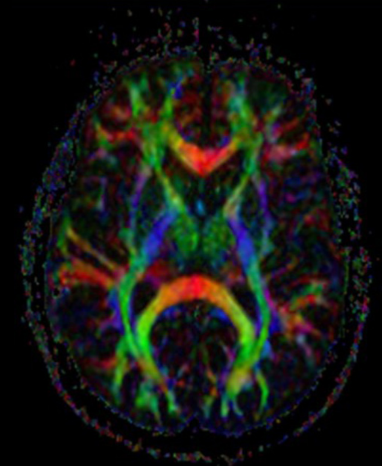
MPRAGE
- T1
- Inflow effects



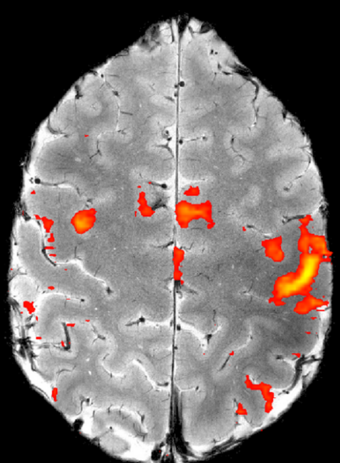
TSE
- T2
- Magnetization Transfer



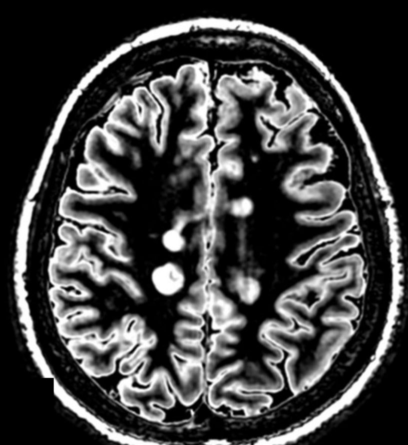
DTI
- Diffusion



BOLD fMRI
- T2* or T2



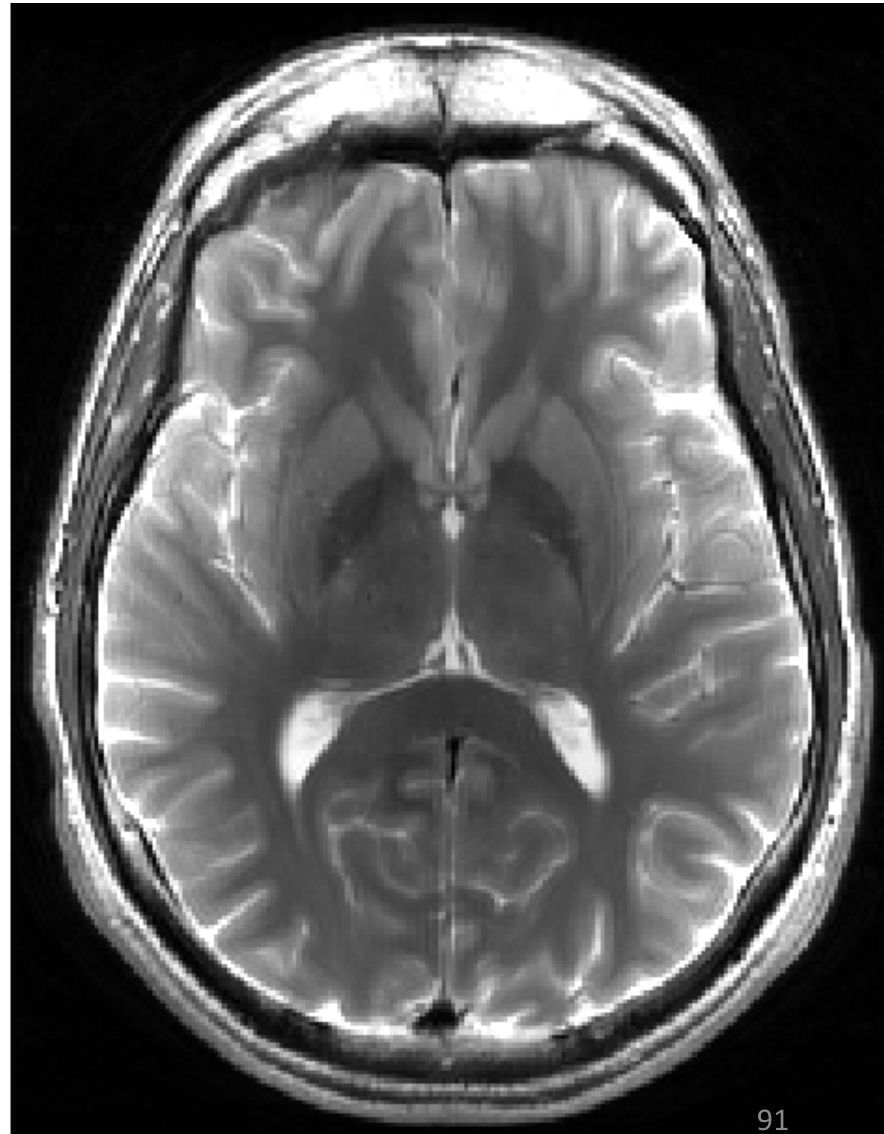
DIR
- T1



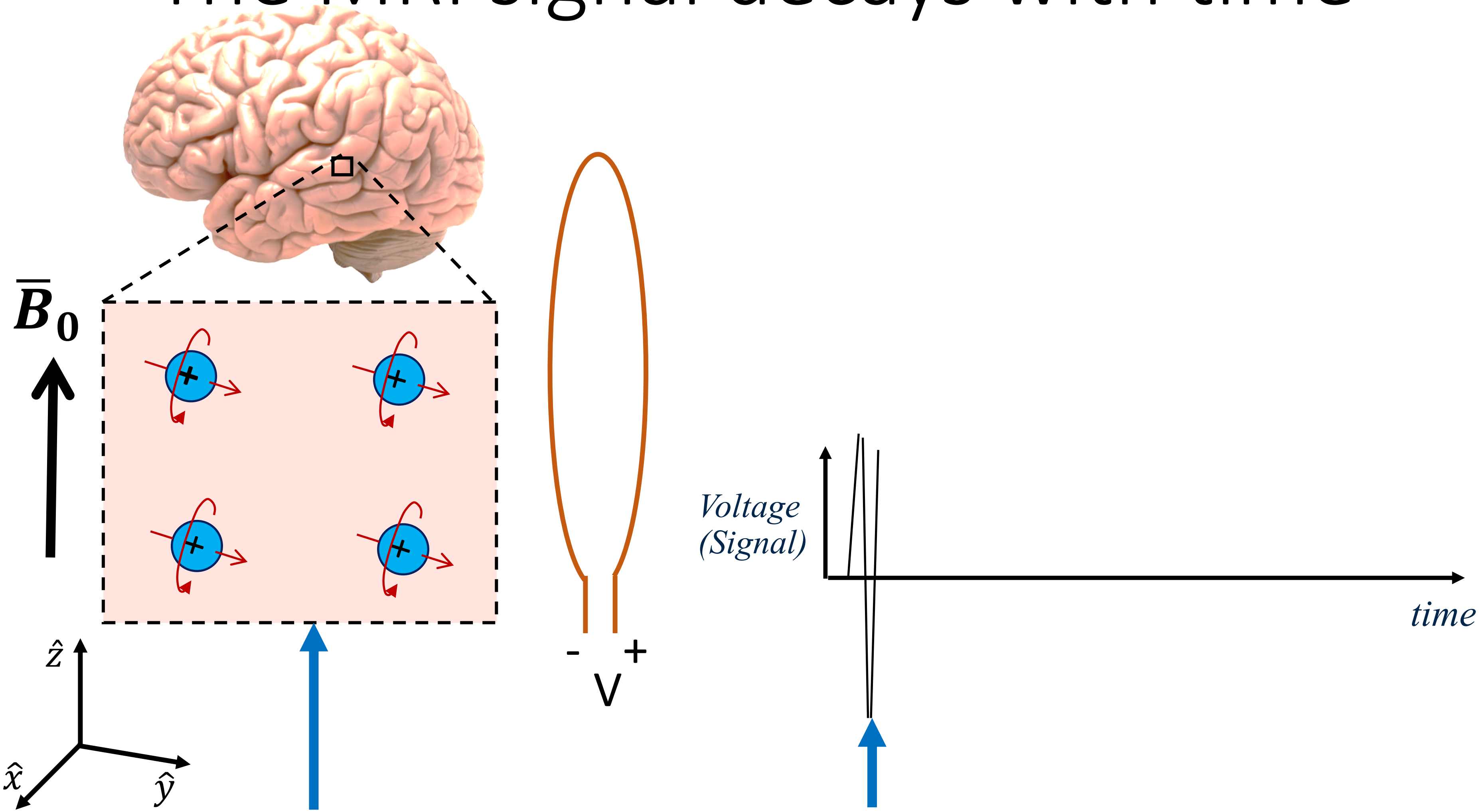
SWI
- T2*
- Magnetic Susceptibility

T_2 Decay

- T_2 varies between tissues
  - Source of image contrast
  - "T_2-weighted" images are 80% of all MRIs
- T_2 decay is irreversible



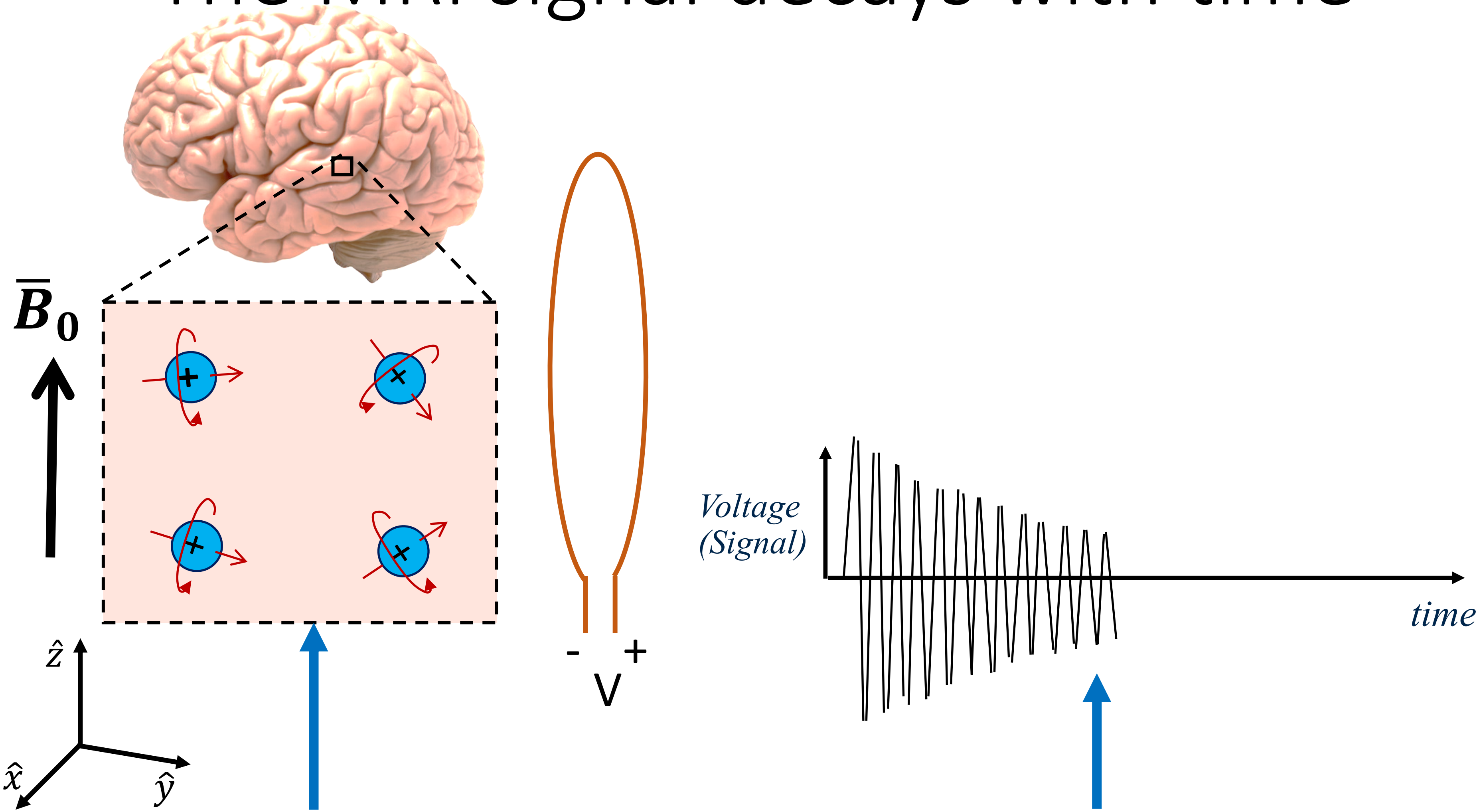
The MRI signal decays with time



Protons precessing together

High-amplitude signal

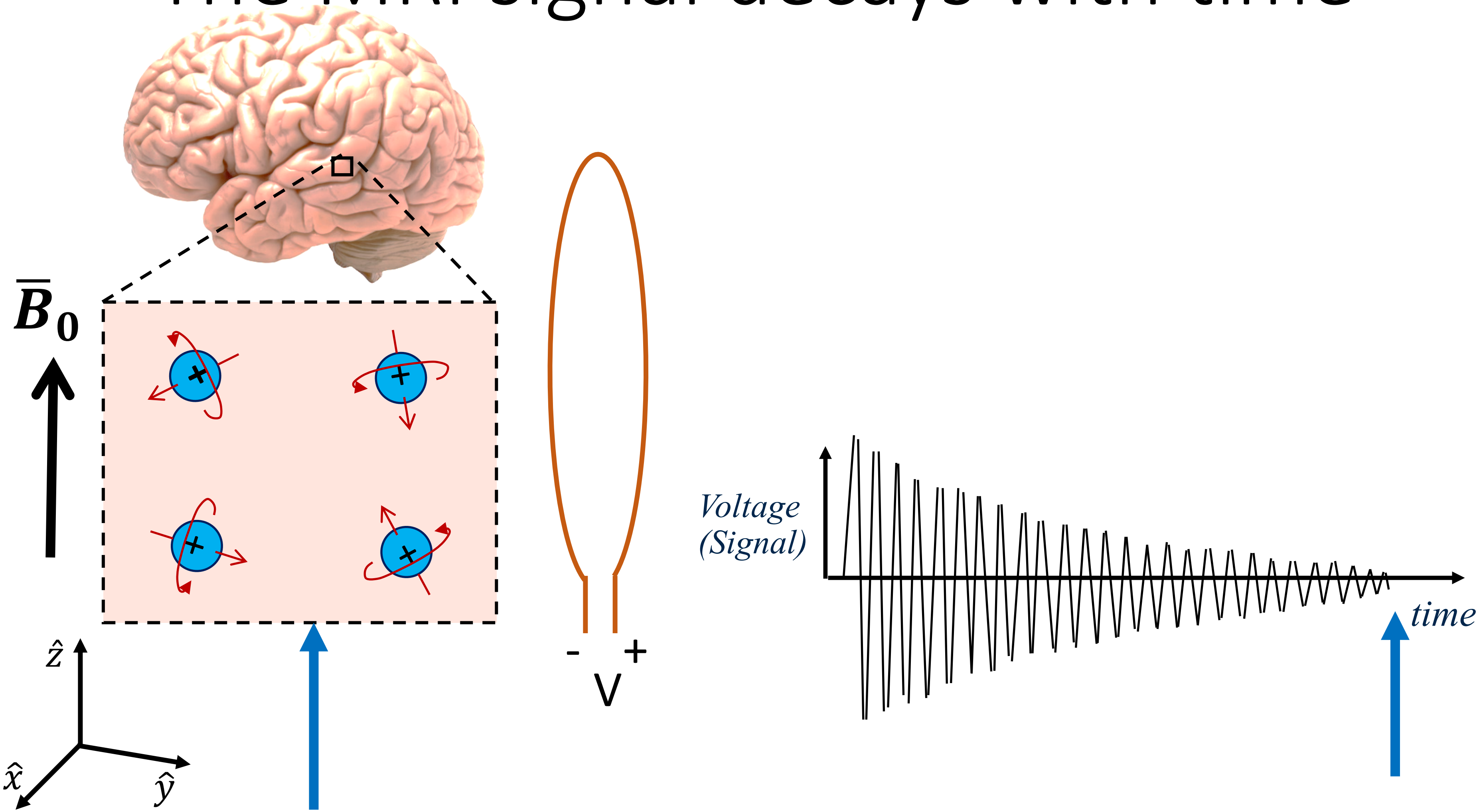
The MRI signal decays with time



Random fluctuations cause misalignment -"dephasing"

Signal amplitude decays

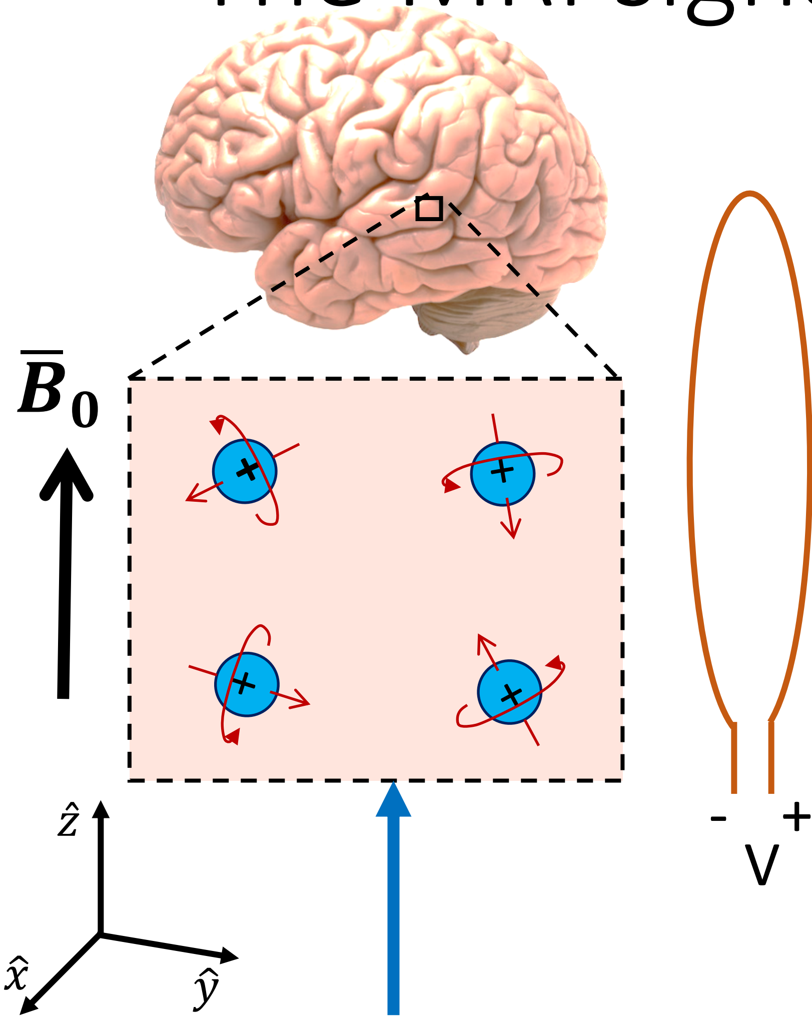
The MRI signal decays with time



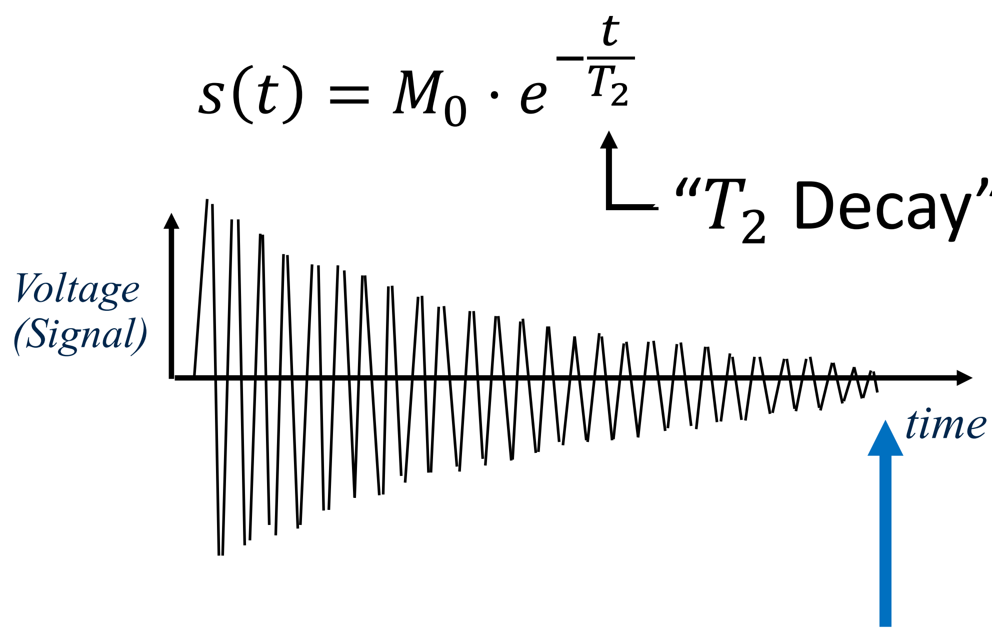
No more coherent proton alignment

Signal has completely decayed

The MRI signal decays with time

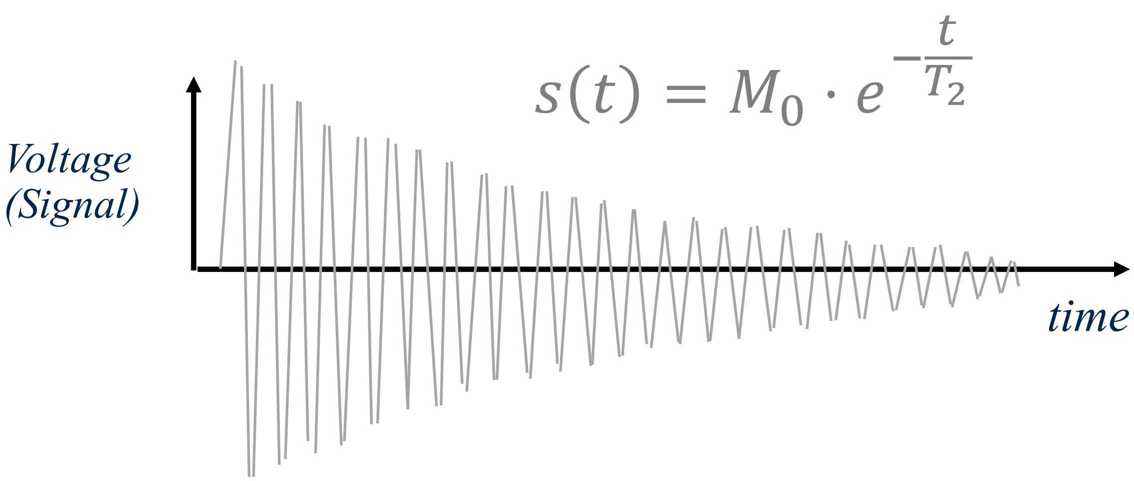


No more coherent proton alignment



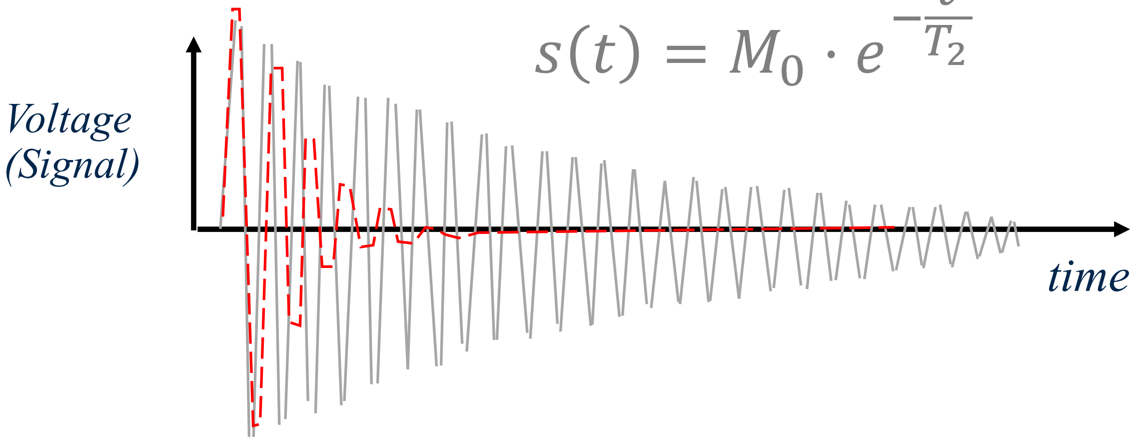
Signal has completely decayed

T_2^*: Signal can decay faster than T_2



T_2^*: Signal can decay faster than T_2

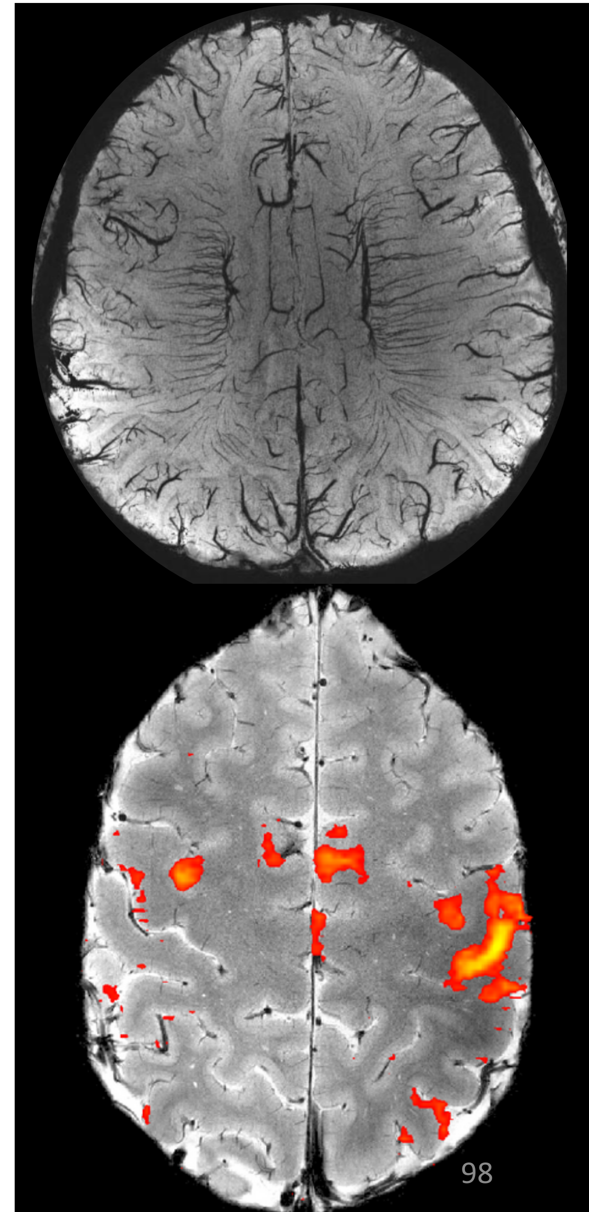
$$s(t) = M_0 \cdot e^{-\frac{t}{T_2^*}}$$



$$s(t) = M_0 \cdot e^{-\frac{t}{T_2}}$$

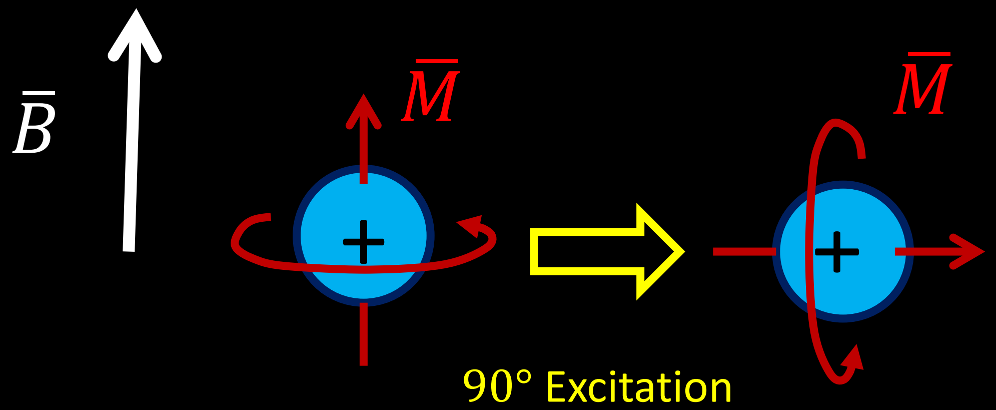
T_2^* Decay

- T_2^* varies spatially and temporally
  - Source of image contrast
  - BOLD effect/fMRI*
- Originates in magnetic materials
  - Air (sinus cavity)
  - Bone
  - Metal (stainless steel retainer).
  - Non-uniform magnet
- T_2^* decay is **reversible**
  - **Not due to random fluctuations**

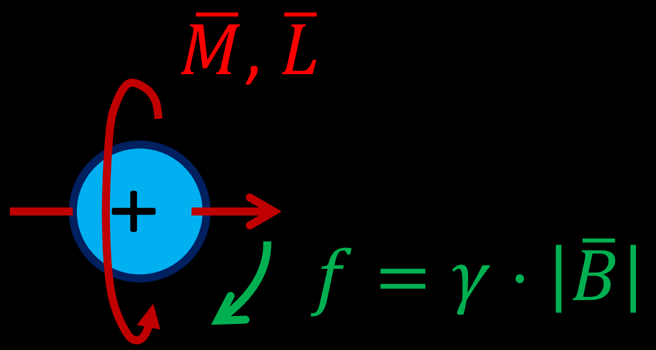


2. Signal Excitation+Detection

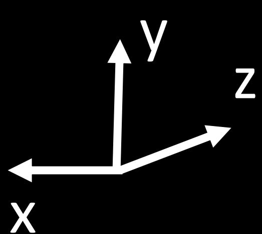
How do we excite spins?

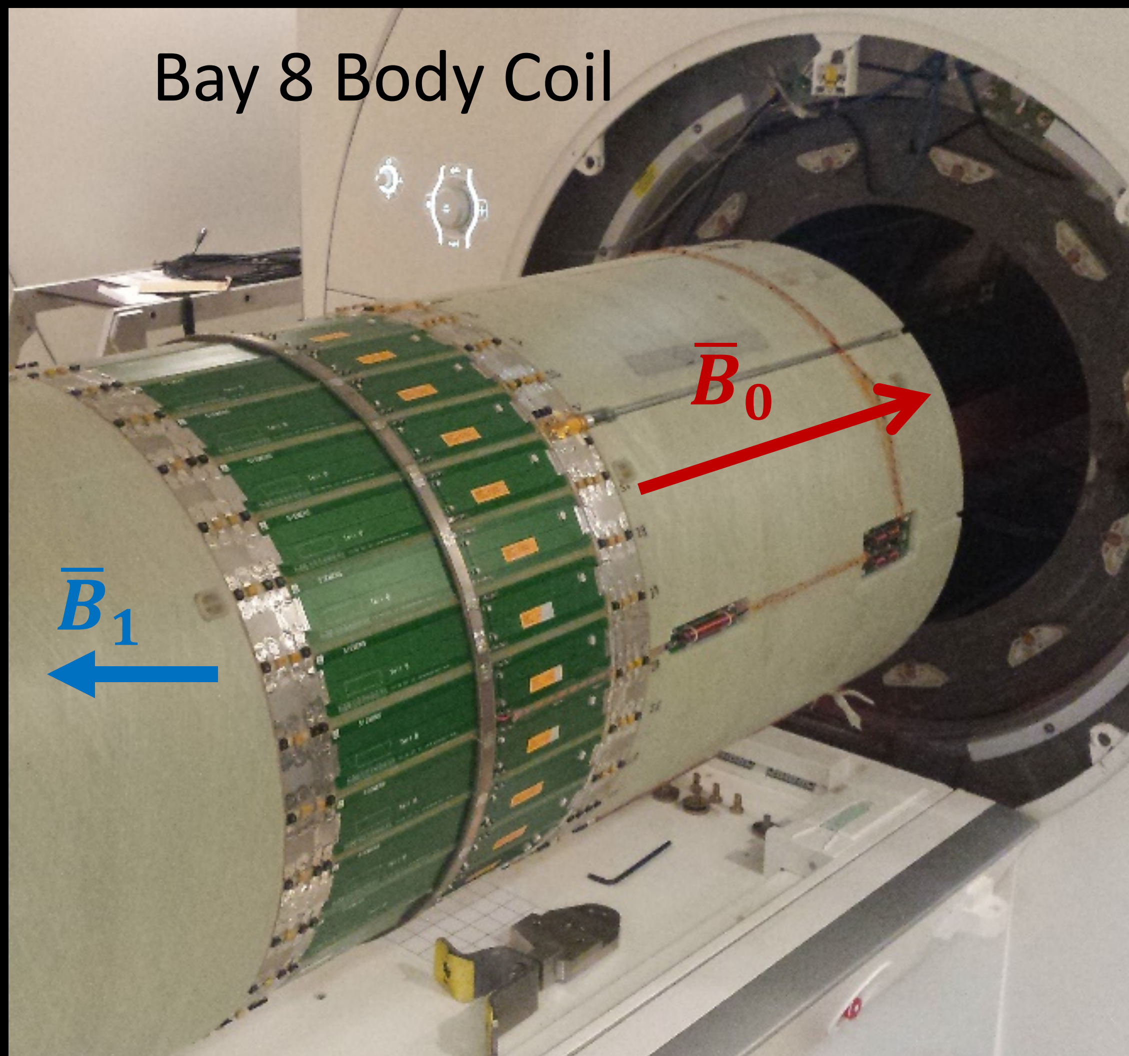


How do we detect precessing spins?

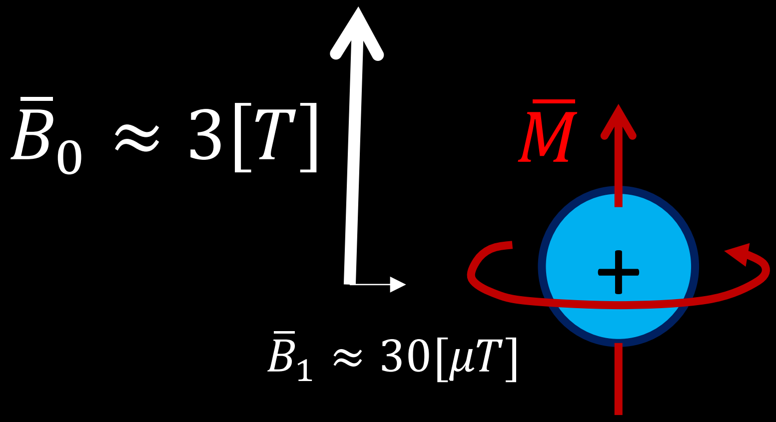


2. Signal Excitation

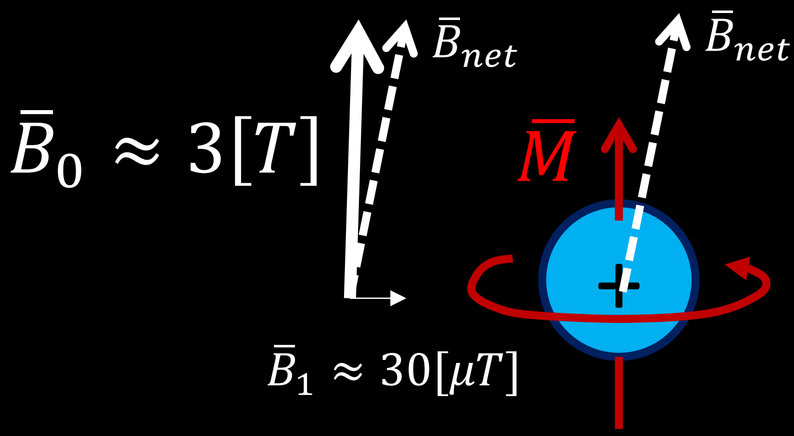




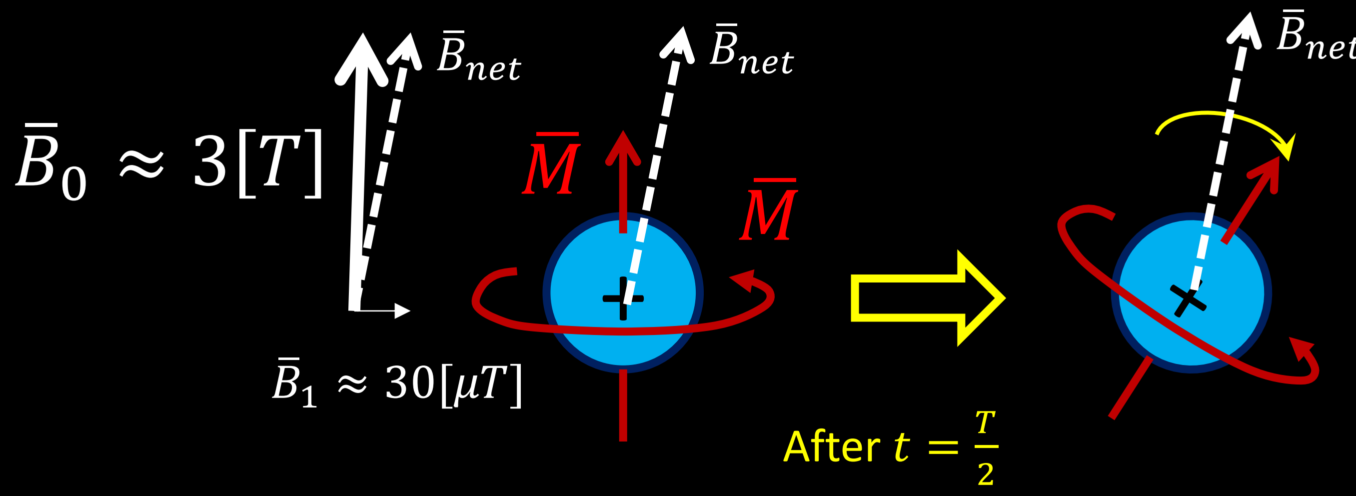
2. Signal Excitation



2. Signal Excitation



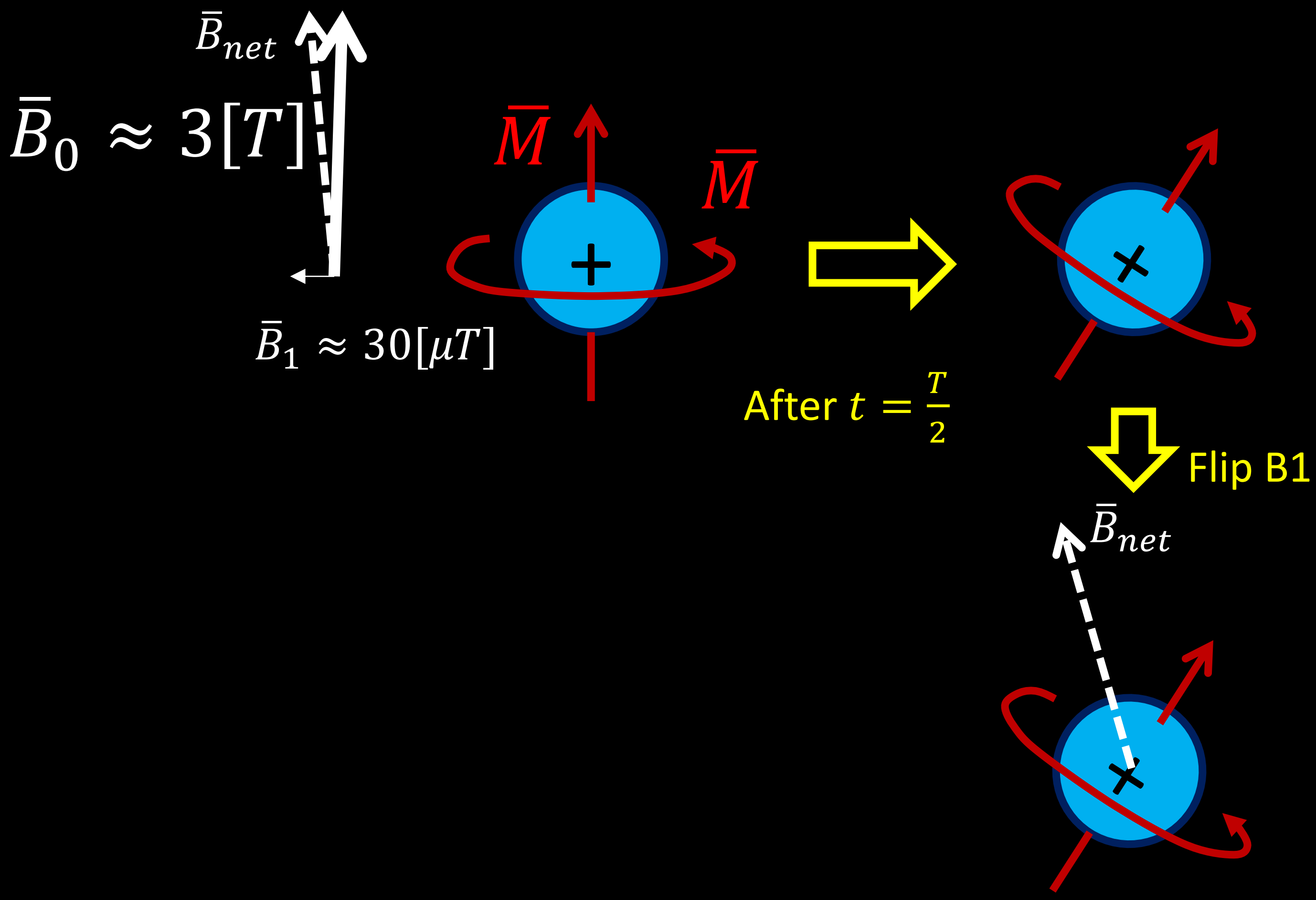
2. Signal Excitation



Larmor Frequency: $f_l = \gamma \cdot B_0 \approx 127.7[MHz]$

Precession Period: $T = \frac{1}{f_l} = 7.8[\text{ns}]$

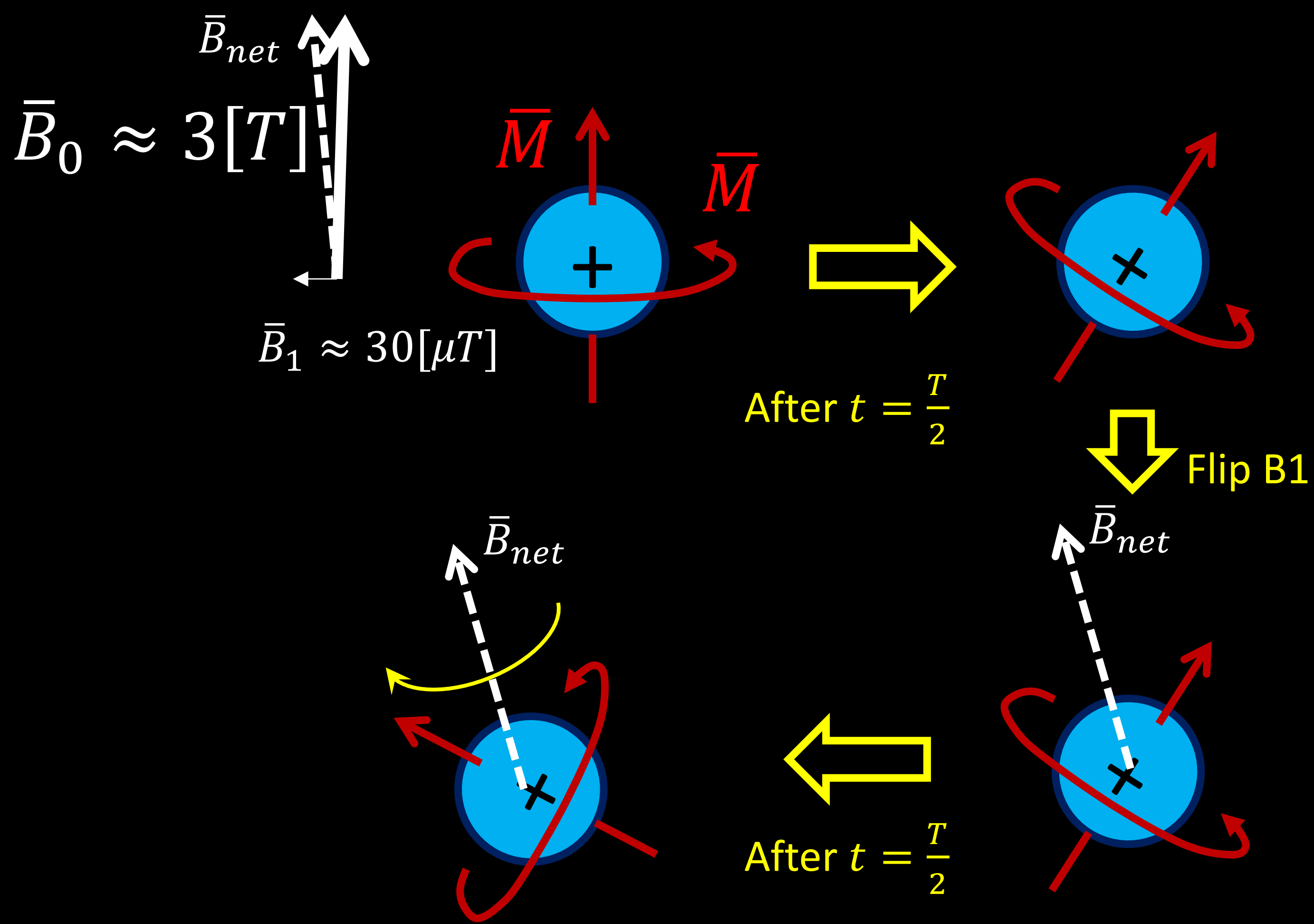
2. Signal Excitation



Larmor Frequency: $f_l = \gamma \cdot B_0 \approx 127.7[MHz]$

Precession Period: $T = \frac{1}{f_l} = 7.8[\mathrm{ns}]$

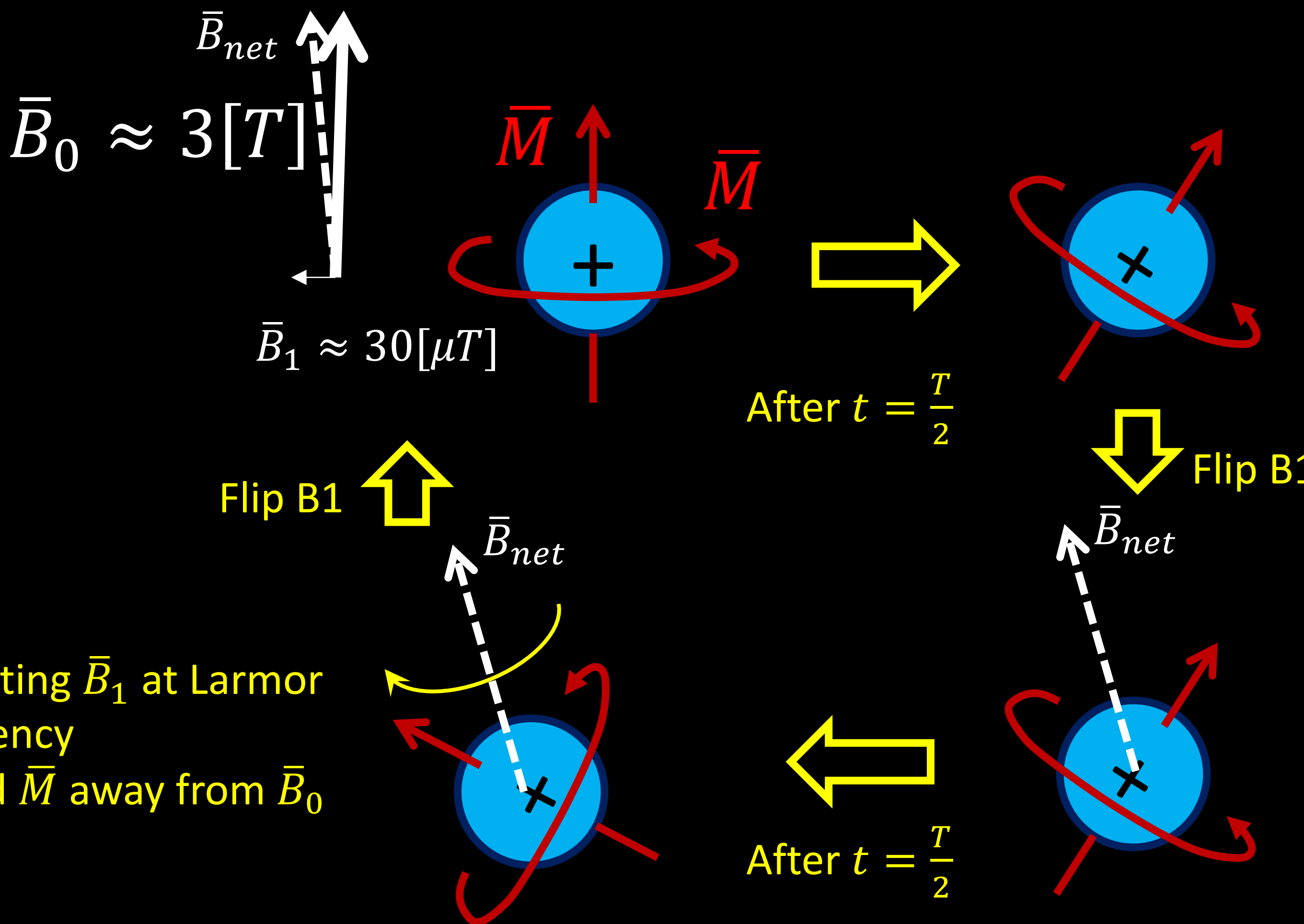
2. Signal Excitation



Larmor Frequency: $f_l = \gamma \cdot B_0 \approx 127.7[MHz]$

Precession Period: $T = \frac{1}{f_l} = 7.8[\mathrm{ns}]$

2. Signal Excitation



- Oscillating $\bar{B}_1$ at Larmor Frequency
- Tipped $\bar{M}$ away from $\bar{B}_0$

Larmor Frequency: $f_l = \gamma \cdot B_0 \approx 127.7[MHz]$

Precession Period: $T = \frac{1}{f_l} = 7.8[\text{ns}]$